

---

# THE EFFECTS OF TIER 2 INTERVENTION ON THE MATHEMATICS PERFORMANCE OF FIRST-GRADE STUDENTS WHO ARE AT RISK FOR MATHEMATICS DIFFICULTIES

---

*Diane Pedrotty Bryant, Brian R. Bryant, Russell M. Gersten, Nancy N. Scammacca, Catherine Funk, Amanda Winter, Minyi Shih, and Cathy Pool*

---

**Abstract.** Responsiveness to Intervention (RtI) is recommended both as an essential step before identifying learning disabilities (LD) and as a mechanism for preventing learning difficulties. The use of evidence-based multi-tiered interventions is of critical importance when implementing RtI. This article presents the results of a study that examined the effects of Tier 2 intervention on the performance of first-grade students who were identified as at risk for mathematics difficulties. Participants included 161 (Tier 2,  $N = 42$ ) first graders. Tier 2 students received 20-minute intervention booster lessons in number and operation skills and concepts for 23 weeks. Results showed a significant intervention effect on the Texas Early Mathematics Inventories-Progress Monitoring (TEMI-PM, University of Texas System/Texas Education Agency) total standard score.

---

*DIANE PEDROTTY BRYANT, Ph.D., The University of Texas at Austin.*

*BRIAN R. BRYANT, Ph.D., The University of Texas at Austin.*

*RUSSELL M. GERSTEN, Ph.D., Instructional Research Group.*

*NANCY N. SCAMMACCA, Ph.D., The University of Texas at Austin.*

*CATHERINE FUNK, M.S., The University of Texas at Austin.*

*AMANDA WINTER, B.A., The University of Texas at Austin.*

*MINYI SHIH, Ph.D., The University of Texas at Austin.*

*CATHY POOL, B.S., The University of Texas at Austin.*

---

There is a growing interest in early mathematics difficulties, stemming in part from prevalence figures indicating that 5% to 10% of school-age children exhibit mathematics disabilities (L. Fuchs, Fuchs, & Hollenbeck, 2007; Gross-Tsur, Manor, & Shalev, 1996; Ostad, 1998). The reauthorization of the Individuals with Disabilities Education Improvement Act (2004) supports the use of Response to Intervention (RtI) as a way of identifying

students with learning disabilities (LD), including students who may have LD in mathematics. Initially conceptualized by Heller, Holtzman and Messick (1982), and further developed by Fuchs and Fuchs (1998), Fuchs, Fuchs, and Speece (2002), and Vaughn and Fuchs (2003), RtI holds promise as an alternative to more traditional approaches to LD identification and as a means to improve procedures associated with prevention and

---

remediation (e.g., implementation of validated practices and assessment of student response to treatment).

Briefly, the RtI approach is characterized by (a) a high-quality general education program that includes universal screening procedures to identify students at risk for academic difficulties, (b) secondary intervention consisting of a standard, evidence-based treatment protocol with progress monitoring for a specified duration, and (c) tertiary intervention that is more intensive and tailored to individual student needs (Fuchs, Mock, Morgan, & Young, 2003; Vaughn & Fuchs, 2003).

Tier 1 is characterized by implementation of evidence-based core instruction for all students (Chard et al., 2008; L. Fuchs, Fuchs, Yazdian, & Powell, 2002). Tier 2 includes intervention to prevent further mathematics difficulties with ongoing progress monitoring to assess response to treatment for students who are identified with risk status in early mathematics skills and concepts. In mathematics, Tier 2 intervention consists of small-group, explicit and systematic instructional procedures incorporating concrete-representation-abstract sequences (Miller & Hudson, 2007) with a fixed duration of instruction. Tier 3, or tertiary instruction, is reserved for students who are struggling to the extent that they require more intensive intervention than a small-group session conducted in their classroom 3-5 days a week.

To date, a multi-tiered prevention and intervention model for operationalizing RtI has been applied in early reading (e.g., Vaughn, Linan-Thompson, & Hickman-Davis, 2003) and, to some extent, in early (primary level) mathematics instruction (D. Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; L. Fuchs et al., 2007). More research is needed in early mathematics (Chard et al., 2005; L. Fuchs et al., 2005; Gersten, Jordan, & Flojo, 2005).

Measures for screening and progress monitoring are increasingly available for schools (e.g., B. Bryant, Bryant, Gersten, Wagner, Roberts, Kim et al., 2008; Chard et al., 2005; L. Fuchs et al., 2007; VanDerHeyden, Witt, Naquin, & Noell, 2001) to identify students at risk. An emerging body of research on young children's mathematics cognition and the way they learn early mathematics concepts is contributing to our understanding of the early numeracy skills that prove problematic for students at risk for mathematics disabilities and should serve as the core of screening measures (Fuchs & Fuchs, 2001; L. Fuchs et al., 2005; L. Fuchs et al., 2007; Geary, Hamson, & Hoard, 2000; Jordan, Kaplan, & Hanich, 2002; Jordan, Kaplan, Oláh, & Locuniak, 2006).

Research results have indicated that students with early mathematics problems exhibit difficulties understanding number sense as demonstrated in number

knowledge and relationships activities (e.g., magnitude, sequencing, base ten) (Jordan et al., 2006); solving word problems (L. Fuchs et al., 2007); and using efficient counting and calculation strategies (e.g., counting on, doubles + 1) to solve arithmetic combinations (i.e., number facts) (D. Bryant et al., in press; L. Fuchs, Fuchs, Hamlet, Powell, Capizzi, & Seethaler, 2006). Findings from studies in these areas informed the design of the preventive intervention practices described in this article, specifically in the area of number sense (number knowledge and relationships, base ten) and arithmetic combinations.

### **Number Sense**

For young students, developing number sense of mathematical concepts and mastery and fluency with arithmetic combinations is critical. *Number sense* is defined as "moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value" (National Council of Teachers of Mathematics [NCTM], 2000, p. 79). Gersten and Chard (1999), Gersten et al. (2005), and Okamoto and Case (1996) further operationalized number sense as the ability to understand the magnitude of numbers, the ability to use representations, and ease of use with mental computation.

**Number sense components.** Jordan et al. (2006) identified a broader array of number sense components in their kindergarten assessment battery, including counting (e.g., counting sequence, counting principles); number knowledge (e.g., quantity discrimination); number transformation (e.g., addition and subtraction verbal and nonverbal calculations); estimation (e.g., of group size using reference points); and number patterns (e.g., extending number patterns, discerning numerical relationships).

According to Jordan et al., these skills relate to the primary-level mathematics curriculum and have been validated as important for developing early mathematics concepts in young children (e.g., Griffin, 2004; Griffin & Case, 1997). For example, studies have shown that many children enter kindergarten understanding counting principles, such as one-to-one correspondence and the cardinality principle, and acquire more advanced counting skills (e.g., counting backwards, counting objects in groups, counting by 10s) in the primary grades (Gelman & Gallistel, 1978; Jordan et al., 2006).

However, young students with mathematics problems have difficulty with the conceptual understanding of some counting principles (e.g., order irrelevance), and counting difficulties affect the use of more advanced counting abilities (e.g., counting on:  $8 + 2 = 11$ ) to

---

solve arithmetic combinations (Case & Okamoto, 1996; Geary, 2004; Griffin, 2004). Number knowledge represents the ability to understand the concept of quantity; that numbers have magnitude and that this magnitude relates to a counting sequence. Importantly, number knowledge has been linked to arithmetic achievement in first grade (Baker et al., 2002). Students use their understanding of number knowledge to develop a “mental number line” (i.e., linear increases of magnitude) to solve calculations “in their heads” and to comprehend place value (Jordan et al., 2006; Siegler & Booth, 2004). Thus, students begin to integrate their conceptual understanding of counting with quantity (Griffin, 2004).

**Importance of place value.** Conceptual understanding and conceptual proficiency for whole numbers – the base-ten system (i.e., place value, computation) – is an important component of mathematics instruction that students must fully grasp (Van de Walle, 2004). Place value understanding can be developed by building connections between important features of instruction, such as grouping objects by 10 and units and using written notations (e.g., numerals) to convey information about the groupings (e.g., 3 groups of 10 and 4 units = 34) (Hiebert & Wearne, 1992).

According to Ross (1989), there are five levels of place value understanding, as follows.

- **Single numeral:** Individual digits in numerals such as 52 are not understood as representing specific values in the number. Instead, 52 is merely a single numeral.
- **Position names:** The student can name the position of the digits, for example, in 52, 5 is in the tens place and 2 is in the ones place, but does not associate value with the position.
- **Face value:** Each digit is taken at face value. In 52, the student selects 5 blocks to make up the 5 and 2 blocks make up the 2. The value of the position is not understood.
- **Transition to place value:** In 52, 2 blocks are selected for the ones place and the remaining 50 blocks are selected for the 5; no grouping of tens is demonstrated.
- **Full understanding:** In 52, 5 groups of 10 are selected, and 2 remaining blocks are chosen for the 2.

Unfortunately, evidence suggests that students do not learn place value concepts sufficiently to understand procedures for multi-digit calculations. Consequently, some students solve computational problems correctly but lack the conceptual understanding of what they are doing (Fuson, 1990).

Jordan, Hanich, and Kaplan (2003) conducted a longitudinal study of 180 students in second grade and fol-

lowed them to third grade. Students were administered a battery of tests designed to assess performance on a variety of early mathematics tasks that included place value. Place value tasks involved problems with standard (e.g.,  $43 = 4$  tens and 3 ones) and nonstandard (e.g.,  $43 = 3$  tens and 13 ones) place value and digit representations (e.g., 43: show with concrete models what 4 stands for; count out 40 chips). Jordan et al. found that over time students with mathematics difficulties scored lower on place value tasks than average students.

These findings suggest that students with mathematics difficulties require sustained instructional time in place value concepts beginning in the early grades and continuing throughout the school years. Ideally, this would comprise much more time in their core (Tier 1) mathematics instruction. In any case, it should become a key component of Tier 2 instruction at this grade level.

Additionally, we know that fluency with basic arithmetic combinations is a challenging area for students at risk in early mathematical skills and concepts. Therefore, Tier 2 intervention must include systematic instruction in addition and subtraction strategies (Geary, 2004).

### *Arithmetic Combinations*

The notion of combining and partitioning groups of objects emerges informally (i.e., experientially) in young children before formal education begins. For young students, instruction in addition/subtraction combinations through problem solving, counting strategies, properties (e.g., associative property), and fact families (i.e., related facts [ $5 + 4$ ,  $4 + 5$ ,  $9 - 4$ ,  $9 - 5$ ]) is a common requirement in states’ mathematics standards (California State Board of Education, 2007; Massachusetts Department of Education, 2000; Texas Education Agency, 2006), typically beginning in kindergarten or first grade. Through opportunities to identify and solve arithmetic combinations, many children learn the “basic facts” and can fluently retrieve solutions to problems.

For students in the early grades with identified mathematics difficulties, this is not the case, however. Numerous studies (D. Bryant et al., in press; Fuchs et al., 2006; Geary, 2004) have documented the difficulties these students manifest in using counting strategies (e.g., counting on from larger:  $5 + 4 = 5 . . . 6, 7, 8, 9$ ; counting down from:  $10 - 3 = 10 . . . 9, 8, 7$ ); retrieval (i.e., recall of fact answer from long-term memory); and derived or decomposed (i.e., recall a partial sum and then count on [ $4 + 5$  can be thought of as  $4 + 4$  plus one more]) strategies efficiently and effectively (see Carpenter & Moser, 1984, and Geary, 2003, for a more

---

complete description of addition and subtraction strategies).

Difficulties with arithmetic combinations have been documented as early as in the first grade (Geary, 1990; Geary et al., 2000) and are pervasive in timed and untimed conditions in subsequent years (Jordan et al., 2003). For example, in a study of first graders with arithmetic disabilities (AD) on the use of strategies to solve addition problems, Geary (1990) found that students with AD did not differ significantly from typically achieving students in using a variety of strategies (e.g., counting on fingers, verbal counting, retrieval). However, they made significantly more errors in retrieval and the counting-on strategy than the typically achieving peer group. Analyses of reaction time for solving facts and other numerical processes also revealed that students with AD exhibited variability in fact retrieval speed, suggesting difficulties with the way facts were represented in long-term memory (Geary, 2003).

Further, Jordan et al. (2003) conducted longitudinal studies involving second- and third-grade students with and without mastery in arithmetic combinations. Over a two-year period, students in both groups progressed at the same rate in untimed conditions. Yet, students with difficulties continued to use their fingers to count on, whereas students in the other group used verbal counting with and without fingers. In the timed condition, students with poor mastery of arithmetic combinations made little progress in fluency, suggesting that deficits in the retrieval of arithmetic combinations continue to be problematic.

In sum, students with difficulties in mastering arithmetic combinations demonstrate immature counting strategies (e.g., counting all, counting on fingers), which contributes to difficulties in developing computational fluency. Moreover, difficulties with arithmetic combinations have been identified as a defining feature of students with mathematics difficulties (Gersten et al., 2005; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan et al., 2002). Thus, it stands to reason that teaching efficient and effective strategies to enhance mastery and fluency of arithmetic combinations should be part of an intervention for students at risk for mathematics disabilities beginning in the early grades.

As in early reading instruction, first grade is a critical year for the development of early numeracy knowledge and understanding. Building on mathematical numeracy taught in kindergarten, first grade is important for continuing to teach young students the skills and concepts that serve as a foundation for later mathematical understandings. Identifying students with risk status for mathematics difficulties is a good first step with measures that are valid in the identification process.

Equally important is implementing valid interventions that can help students learn early numeracy skills, relationships of 10, and basic calculations that can be further developed and built upon in later years. Thus, the purpose of this study was to determine the effects of Tier 2 intervention booster lessons on specific number, operation, and quantitative reasoning performance of students in first grade who were identified as having mathematics difficulties.

## METHOD

### *Participants*

This study was conducted in a primary school that included pre-kindergarten through second grade, located in a suburban school district in central Texas. Participants consisted of 161 (Tier 2,  $N = 42$ ) students in first grade for whom signed consent forms were obtained. Students participated in both pre- and posttest assessments. This group of students was part of a larger sample that included additional students and grade levels (see D. Bryant et al., 2008). The subsample of students in this study was chosen because they attended the same school in which the D. Bryant et al. (in press) study was conducted. In that study, no significant program effect was found for first-grade students. We were interested in seeing the effects of the intervention in the same school a year later, once recommended changes had been made to the first-grade intervention.

Demographic characteristics of the sample were obtained from the school district. Only students whose first language was English were eligible to participate in the study because the measures and intervention were written in English. For the school district, 39.9% of the students were classified as economically disadvantaged based on free/reduced-cost lunch data. In the treatment group, 45.9% of the students were male, 54.1% were female. In the treatment group, 8.1% were African American, 27.0% were Hispanic, 51.4% were White, and 13.5% were Asian/Pacific Islander. In the non-treatment group, 45.3% of the students were male and 54.7% were female. The ethnic breakdown for the non-treatment group was as follows: 12.8% were African American, 29.9% were Hispanic, 35.9% were White, and 21.4% were Asian/Pacific Islander.

### *Design*

A regression-discontinuity design (RDD) was used to determine the effectiveness of the early intervention program. RDD is a quasi-experimental design that is a strong alternative to a randomized experiment when the goal is to evaluate a program's effectiveness (Shadish, Cook, & Campbell, 2002; Trochim, 1984). RDD is appropriate when the group receiving interven-

---

tion and the comparison group are purposely selected to differ in ability prior to implementation of an intervention. Using a quantitative pretest measure, participants for the intervention group are selected using a cut-score criterion whereby only students scoring below the predetermined cut score receive the intervention. Successful implementation of RDD requires that the cutoff criterion is adhered to strictly (i.e., all students falling below the cutoff score receive intervention, and no students at or above the cutoff score receive the intervention). When this condition is met, RDD is a robust alternative to a randomized experiment, which has the added benefit that it does not require researchers to deny intervention to students who need it in order to construct a control group. Strict adherence to the quantitative criterion used to assign students to intervention and comparison conditions minimizes the effect of extraneous variables on study results.

RDD assumes that in the absence of intervention, the relationship between the pretest score (the criterion used to select students for intervention) and the posttest outcome score is the same for all students (those who did and those who did not qualify for intervention). If the intervention had no significant effect, the regression line for pretest and posttest scores would be the same for all students. If the intervention was effective, it would significantly raise the scores of all students who scored below the cutoff and shift the y-intercept of their pre-/post-regression line.

Data analysis examines the degree to which this shift has occurred and determines whether the shift is greater than would be expected by chance alone. A statistically significant discontinuity between these two regression lines indicates a main effect for the program. RD also determines if there is an interaction effect. An interaction effect is present when the intervention is effective only with a subgroup of those who received intervention, such as those who scored particularly high or low on the pretest.

Students were identified to participate in the Tier 2 intervention group based on their total score on the Texas Early Mathematics Inventories: Progress Monitoring (TEMI-PM) measure, which is described in the Measures section. The total score, derived by summing the four subtests, is the most reliable index to use for identification purposes. Students who scored below the 25th percentile (total standard score below 90) on the TEMI-PM in the fall were assigned to the Tier 2 treatment group. Students who scored at or above the 25th percentile (total standard score of 90 or above) did not receive Tier 2 intervention. All students took the TEMI-PM measure in the fall, winter, and spring.

## Measures

During the academic year 2006-2007, all participating students were administered a set of researcher-designed mathematics measures, the TEMI-PM (2006-2007) in the fall (September), winter (January) and spring (late April/early May). The mathematics subtests from the Stanford Achievement Test-Tenth Edition (SAT-10; Harcourt Assessment, 2003) were also administered in the fall of 2006 and the spring of 2007. In the fall, students were administered the SESAT II level of the SAT-10; in the spring, they were given the Primary I version, which consists of Mathematics Procedures (MP) and Mathematics Problem Solving (MPS). Mathematics Procedures is composed of addition and subtraction items. Mathematics Problem Solving items assess a variety of mathematics skills (e.g., numeration, operations, word problems, statistics and probability). A Total Mathematics Score (TMS) is the sum of the Mathematics Procedures and Mathematics Problem Solving scales. The SAT-10 internal consistency reliability coefficients for our sample were computed using the coefficient alpha technique (fall, SESAT 2: .88; spring, Primary 1-MP: .84, MPS: .88, TMS: .91). The concurrent validity of the TEMI-PM scores was assessed by correlations with the SAT-10 and is reported by subtest and total score below.

The TEMI-PM consists of four forms (A, B, C, D). There are four subtests: Magnitude Comparisons, Number Sequences, Place Value, and Addition/Subtraction Combinations. An aggregate total score of the four subtests was used to measure pre-/post student performance in the RD analysis because it is the most robust indicator of performance of the four constructs (B. Bryant, Smith, & Bryant, 2008). Descriptions of the TEMI-PM subtests and the total score are provided below. Included at the end of each test description are data concerning reliability and validity. These data provide beginning evidence for the construct validity of the TEMI-PM scores reported in this study.

**Magnitude comparisons (MC):** This subtest assesses a child's ability to differentiate the smaller of two numbers that are shown side-by-side within a box. The measure is similar to that used by Clarke and Shinn (2004) and Chard et al. (2005) in their Quantity Discrimination measure.

When taking this test, students look at two numbers that appear side-by-side in a box in their student booklet (a vertical dotted line separates the two numbers). Numbers range from 0 through 99. As a fluency measure, the test is designed to determine how many items the student can answer correctly in 2 minutes by circling the smaller of the two numbers or circling both numbers if they are the same (equal). The number of correctly identified numbers constitutes the raw score.

---

Immediate test-retest reliability with alternate forms coefficients for Form A with Forms B, C, and D ranged from .70 to .78 (median = .74). Correlating the spring Form A MC score with the Total Mathematics score from the SAT-10 yielded a concurrent validity coefficient of .64.

**Number sequences (NS):** This measure assesses a child's ability to identify a missing number from a sequence of three numbers. The missing number could appear in any of three positions: the first, second, or third number. This measure is adapted from one used by Clark and Shinn (2004) in their Missing Number test.

In taking this test, students look at a three-number sequence in which one number of the sequence is missing and is represented by a blank. (The missing number may be the first number in the sequence, the second number, or the third number.) The student then looks at four possible response choices in boxes below the stimulus series and circles an answer from among the four response choices. (Numbers range from 0 through 99.) As a fluency measure, the test is designed to see how many items the student can answer correctly in 2 minutes. The amount of correctly identified missing numbers is summed to constitute the raw score.

Immediate test-retest with alternate-forms reliability coefficients for Form A with Forms B, C, and D ranged from .74 to .82 (median = .76). Spring intercorrelations between Form A Number Sequence scores and SAT-10 Total Mathematics score resulted in a concurrent validity coefficient of .60.

**Place value (PV):** This test is designed to assess first graders' knowledge of place value. The test uses a format similar to what is commonly seen in early mathematics textbooks (e.g., Scott Foresman-Addison-Wesley, Charles et al., 1999; Science Research Associates/McGraw-Hill, Bell et al., 2001). Values on the scale range from 1 to 99.

When taking this test, students look at pictures of stacks of tens and individual ones up to 99, look at four possible response choices in boxes below the stimulus item, and then circle the number that shows "how many." As a fluency measure, the test is designed to determine how many items the student can answer correctly in 2 minutes.

Intercorrelation coefficients between Form A and Forms B, C, and D ranged from .67 to .77 (median = .71). Correlating Form A with the Total Mathematics score from the SAT-10 yielded a validity coefficient of .58.

**Addition/subtraction combinations (ASC):** This measure assesses students' ability to correctly write the answers to addition and subtraction facts (sums or minuends ranging from 0-18). When taking this test, students look at addition and subtraction problems on a

page and then compute and write the answer to each problem. As a fluency measure, the test is designed to determine how many items the student can answer correctly in 2 minutes. The total number of correctly computed problems written correctly (e.g., with no reversals or gross illegibility) is summed to produce the raw score. Immediate test-retest with alternate-forms reliability coefficients for Form A with Forms B, C, and D ranged from .78 to .86 (median = .80). We correlated the results of Form A Addition/Subtraction Combinations with the Total Mathematics score from the SAT-10, which resulted in a validity coefficient of .68.

**Total score (TOT):** The Total Score was derived by summing the raw scores for MC, NS, PV, and ASC. Immediate test-retest with alternate-forms reliability coefficients for Form A with Forms B, C, and D ranged from .83 to .88 (median = .86). We also correlated the spring Form A Total Score with the Total Mathematics score of the SAT-10; the resulting concurrent validity coefficient was .72.

### **Data Collection Procedures**

During testing, project staff administered the tests in intact classrooms to students who had returned signed affirmative permission slips. Classroom teachers were present during testing to help keep students on task and for behavior management purposes. Timers were used to ensure accuracy in the amount of time allotted for testing.

A 3-hour training session on all measures was conducted in late summer 2006 for project testers, who were either project staff members or undergraduate or graduate students in general education, special education, or educational psychology. All testers had taken a basic assessment course.

During training, administration and scoring procedures for each of the measures and the SAT-10 were reviewed and modeled. Modeling was followed by a question-and-answer period, whereupon prospective testers were paired with testers ("veteran" testers) from previous years to practice giving the tests using scripted directions. Veteran testers provided feedback until the new testers were comfortable and accurate in test administration. Refresher trainings were conducted in the winter and spring.

During the actual test administration in the nine first-grade classrooms, veteran testers were initially paired with new testers to conduct observations of test administration and provide feedback about the procedures. Then, all testers conducted the assessment with only the classroom teachers present.

### **Tier 2 Intervention Booster Lessons**

Based on the findings from our previous study (D. Bryant et al., 2008), which included first-grade students,

---

we recommended that the tutoring sessions be increased to allow more time for delivery of the intervention booster lessons. Thus, tutoring sessions occurred for four days per week for 20 minutes per session across 23 weeks as opposed to three-four days for 15-minute sessions over 18 weeks as in the previous study. A period of 23 weeks was selected to allow sufficient time for Tier 2 instruction that paralleled the core curriculum in terms of when topics in number and operation and word problem solving were taught during the school year.

Two tutors taught the Tier 2 intervention lessons. One full-time tutor and one graduate research assistant (GRA) were trained by the first author to implement the lessons. The full-time tutor was a former teacher who had taught kindergarten for 17 years. The GRA was a full-time doctoral student in the Department of Educational Psychology, who tutored 20 hours per week for the project. The GRA had previous teaching and tutoring experience. Both tutors had been with the project for two years and, thus, were considered “veteran” tutors.

**Tutor training.** At the beginning of the program, initial training consisted of (a) an explanation of the program; (b) a description of the lessons; and (c) an explanation of procedures for explicit, systematic instruction. Tutors were given time to practice the initial lessons. Once the program began, additional tutor training consisted of reviewing new lessons and making adjustments to lessons based on tutors’ feedback. Additional training was conducted on upcoming lessons usually on a biweekly basis. This training was conducted for two-hour time segments either after school or during the school day. Ongoing weekly communication about the tutoring was conducted via email.

**Tutoring program.** The Tier 2 intervention program occurred in small groups (4-5 students) with scripted booster lessons. Each instructional session consisted of number, operation, and quantitative reasoning skills and concepts. The content of the booster lessons was based on the Texas Essential Knowledge and Skills (TEKS) number, operation, and quantitative reasoning skills and concepts standards. These skills were the focus of this Tier 2 intervention.

Skills and concepts taught included:

- Counting and Number Sense
- Counting: rote, counting up/back
- Number recognition & writing: 0-99
- Comparing & grouping numbers
- Number relationships of more, less
- Relationships of one and two more than/less than
- Part-part-whole relationships (e.g., ways to represent numbers)
- Numeric sequencing

Place Value/Relationships of 10:

- Making and counting: groups of tens and ones
  - Using base-ten language (2 tens, 6 ones) and standard language (26) to describe place value
  - Reading and writing numbers to represent base-ten models
  - Naming the place value held of digits in numbers
- Addition/Subtraction Combinations (sums and minuends to 18, respectively):
- Identity element and properties
  - Fact families
  - Counting and decomposition strategies (e.g., Addition: count on [+ 1, + 2, +3], doubles [6+6] doubles +1 [6+5], make 10 + more [9+5]; Subtraction: count back/down [-1, -2, -3])

Specific content was designated for instruction over a two-week instructional time period. Instructional emphasis was placed on number concepts that are problematic for students with mathematics difficulties (e.g., teen numbers, difficult facts).

Explicit, systematic teaching procedures and strategic instruction were employed to teach the content. These procedures included brisk pace, opportunities to respond, error correction, and strategies for learning the arithmetic combinations. As part of explicit instruction, tutors modeled the processes or steps necessary to solve problems or provided explanations of how to perform skills. Strategic instruction consisted of teaching students specific strategies for learning addition and subtraction combinations. We also used the concrete-semi-concrete-abstract (CSA) approach to instruction, as appropriate (Butler, Miller, Crehan, Babbit, & Pierce, 2003; Mercer, Jordan, & Miller, 1996). To accompany the CSA approach, we used base-ten models and counters (concrete level), number lines and hundreds charts (for semi-concrete), and manipulation of numerals (abstract level).

A behavior management system was instituted whereby students earned a sticker each day for appropriate “Math Ready” behavior. “Math Ready” behavior included listening to the teacher, responding, and sitting up in one’s chair. Students were given stickers at the end of each tutoring session for appropriate behavior. The stickers were placed on a chart, which was publicly displayed in the tutoring classroom. At the end of each week, students received their stickers. Also, tutors told classroom teachers about the students’ behavior so that students could earn a certificate, which was part of a school-wide behavior management system.

Finally, daily activity-level progress monitoring was conducted. Students were given four either oral or written problems to determine their response to instruction on each booster lesson taught that day. The majority of students in the group had to demonstrate

**Table 1**

***Examples of Booster Lessons for Addition/Subtraction Combinations***

---

**Instructional Booster: Doubles + 1**

Objective: The students will correctly answer the double + 1/related facts.

Strategy: Doubles + 1

Representation: Concrete, Abstract

Vocabulary: Doubles, plus one

**Preview**

We are going to learn how to use our doubles facts to answer double + 1 facts.

**Review**

1. Review the doubles strategy: When you have addition with 2 numbers that are the same, this is a double.
2. Use "Look and Say." Review the doubles for students to answer.

**Modeling (My Turn)**

1. Say and Make, "I have 12 connected cubes. I break them into 2 equal parts. Count with me how many in each (6)."
2. Make, 2 rows of 6.
3. Say, "This is a double fact:  $6 + 6 = 12$ ."
4. Say and Make, "I add one to a 6 part (add to second row of 6). This makes 7. I have a double,  $6 + 6$ , + 1 more,  $6 + 7$ . If  $6 + 6 = 12$ , what is one more than 12? (13). How much is  $6 + 7$  (13)."

5. Write, 6 and 6 on the wipe board (do not erase). Have students read the facts.

$$\begin{array}{r} +6 \\ 6 \\ \hline 12 \end{array} \quad \begin{array}{r} +7 \\ 6 \\ \hline 13 \end{array}$$

6. Say, "Here's the strategy: 2 numbers next to each other on the number line (point to 6, 7 on the number line), take the smaller number (6), think the double ( $6 + 6$ ) and its answer (12), then add +1 to the double answer (13) because 7 is one more than 6 so 13 is one more than 12."

Ask students to repeat the strategy together. Prompt students who need help saying the strategy.

Point out that  $6 + 7$  is a turnaround fact for  $7 + 6$ ; the strategy still works.

**Guided Practice (Our Turn)**

1. Give students 8 cubes to connect together. Count out loud in unison to connect.
2. Have students break the group of 8 into 2 equal groups. Have them make 2 rows of 4 to show that they are equal.
3. Ask what double fact is shown ( $4 + 4 = 8$  - vertically); write it on the wipe board.
4. Have students add 1 to a part of 4. Ask, how many (5). Ask, what is the double + 1 fact ( $4 + 5 = 9$ -vertically). Write it on the wipe board next to  $4 + 4 = 8$ . Remind students that  $5 + 4$  also equals 9 because it is a turnaround fact.
5. Have students repeat the strategy.
6. Repeat steps 1-4 for the remaining doubles + 1 facts.
7. Work with students to complete the 2.GP Doubles + 1 Sheet.

*continued next page*



---

**Table 1 continued*****Examples of Booster Lessons for Addition/Subtraction Combinations***

---

**Practice Booster: Doubles + 1****Preview**

We are going to review our double + 1 facts.

**Review**

1. Review the doubles strategy: When you have addition with 2 numbers that are the same, this is a double.
2. Use "Look and Say." Review the doubles facts for students to answer.

**Guided Practice (Our Turn)**

1. Show students flash cards with facts (some doubles + 1, some doubles). Ask students to do a "thumbs up" if the fact is a doubles + 1 fact. Ask how they know (the fact has 2 numbers that are next to each other on the number line).
2. Lay out the doubles + 1 facts and a number line.
3. Say, "*Here's the strategy: 2 numbers next to each other on the number line (6, 7), take the smaller number (6), think the double (6 + 6) and its answer (12), then add +1 to the double answer (13) because 7 is one more than 6 so 13 is one more than 12.*"
4. Show a doubles fact. Have students tell the answer in unison. Have students tell which doubles + 1 fact it goes with. Have students tell the answer. Have students use the number line and repeat the strategy.
5. Work with students to complete the Doubles vs Doubles + 1 guided practice sheet.

**Independent Practice (Your Turn) Total: 2 minutes**

1. For 1 minute, have students complete IP Doubles + 1 sheet.
2. Then, for 1 minute have students check () each response for correct answer. Conduct error correction as needed.

**Instructional Booster: Doubles + 1 Fact Families****Preview**

We are going to practice doubles + 1 and fact families.

**Review**

1. "What is a fact family?" (3 numbers that go together to make addition and subtraction facts). What is an example?"
2. Review the Doubles + 1 strategy: "*Here's the strategy: 2 numbers next to each other on the number line (6, 7), take the smaller number (6), think the double (6 + 6) and its answer (12), then add +1 to the double answer (13) because 7 is one more than 6 so 13 is one more than 12.*"
3. Use "Look and Say" to review the doubles + 1 facts in unison.

**Modeling (My Turn)**

1. Show  $4 + 5 = 9$  - "What answer?"
2. Show and Say, " $9 - 4 = 5$ ." Point out the minus sign.
3. Explain that  $4 + 5$  and  $9 - 4$  is a fact family; 4, 5, 9 go together in a family.
4. Repeat steps 1 - 3 for  $5 + 4$ , a turnaround fact.
5. Put them in a fact family pack.

*continued next page*

---

**Table 1 continued**

***Examples of Booster Lessons for Addition/Subtraction Combinations***

---

**Instructional Booster: Doubles + 1 Fact Families continued**

**Guided Practice (Our Turn)**

1. Give each student (and the teacher) one fact from a fact family.
2. Put the remaining facts (doubles + 1/related) in a pack. Draw a fact and have students identify who has the other fact for the family.
3. When all cards are distributed, in unison, have students identify the 3 members of each family.
4. Show the doubles + 1/related flash card facts (no answer) and do "Look and Say."

**ERROR CORRECTION:**

Correct the error and put facts that are errors in a pile. Provide more practice.

**Independent Practice (Your Turn)**

1. Have students complete the Doubles + 1 fact +/- sheet for 1 minute.
2. Go through problems with students telling them the correct answers for 1 minute. They should put ✓ if correct; or correct errors

accuracy on three out of four of the problems to consider the lessons successful for each day. Examples of booster lessons for Addition/Subtraction Combinations are presented in Table 1.

***Fidelity of Implementation***

To assess the quality (i.e., fidelity) of the implementation of specific performance indicators, the project coordinator and project consultant observed treatment sessions for each tutor for four sessions during the 23-week intervention. Quality of Implementation (QoI) indicators included the degree to which tutors (a) followed the scripted lessons for the content (e.g., modeling, guided practice, independent practice); (b) implemented the features of explicit, systematic instruction (e.g., pacing, error correction, minimal teacher talks, engagement); (c) managed student behavior (e.g., use of reinforcers and redirection); and (d) managed the lesson (e.g., use of timer, smooth transitions between booster lessons). Performance indicators were rated on a 0-3 point scale, where 0 = Not At All, 1 = Rarely, and 2 = Some of the Time, and 3 = Most of the Time.

Results were shared with the tutors, and areas in need of further training were addressed. Results on the qual-

ity performance indicators were as follows: (a) following the scripted lessons for the content: median of 3.0; (b) implementing the instructional procedures: median of 2.80 with a range of 2.80-3.00; (c) managing student behavior: median of 2.80 with a range of 2.80-3.00; and (d) managing the lesson: median of 3.0. These results across tutors show a high degree of fidelity in the implementation of the booster lessons.

**RESULTS**

Pre- and posttest data were available for 42 students who qualified for the Tier 2 intervention, and for 119 students who scored above the benchmark and thus did not receive intervention (Tier 1 only). The RD analysis demonstrated that a significant main effect ( $\beta = .21$ ,  $p = .014$ ) for the spring TEMI-PM total standard score, indicating a positive effect for the intervention with first-grade students.

Figure 1 depicts a scatter plot of scores and regression lines for Tier 1 and Tier 2 students. There is a discontinuity (gap) at zero on the x-axis between the regression line for the Tier 2 (at-risk) group and the Tier 1 (not-at-risk) group depicting the positive effect of the program on at-risk students. This discontinuity results in a shift

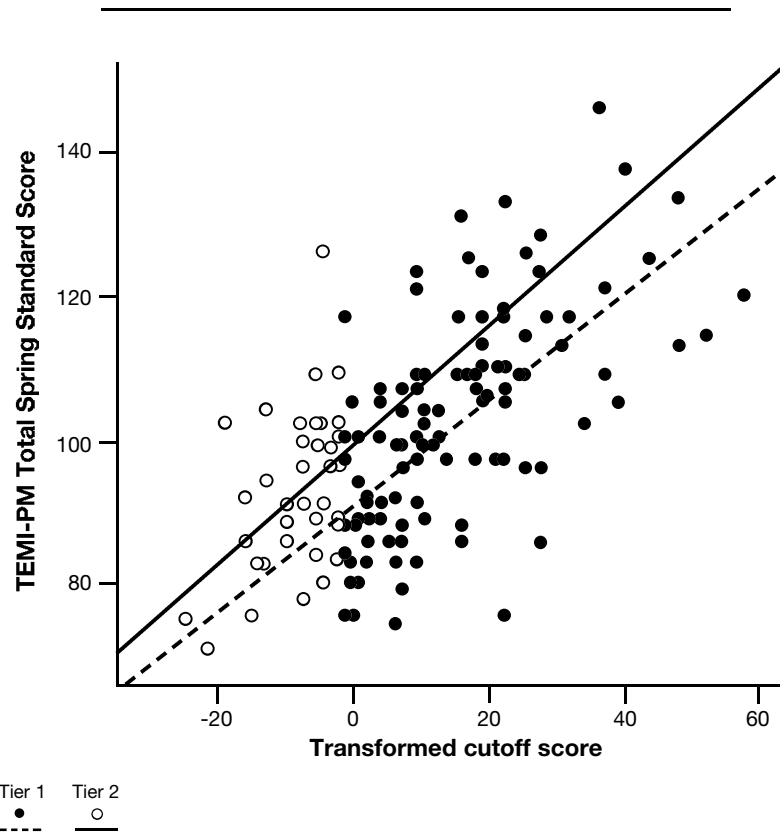
of the y-intercept of the regression line for pre-/post scores for at-risk students above that of the regression line for not-at-risk students. Thus, we concluded that the posttest scores of at-risk students were significantly higher than expected based on their pretest scores, which resulted in the finding of a main effect for the intervention.

At the subtest level, regression discontinuity analyses showed a program effect for the Number Sequences ( $\beta = .19, p = .048$ ) and Addition/Subtraction subtests ( $\beta = .20, p = .029$ ). A significant interaction effect also was found for Magnitude Comparison ( $\beta = .20, p = .028$ ) with Tier 2 students with the lowest scores showing a positive effect. For Place Value, no significant effect was detected.

## DISCUSSION

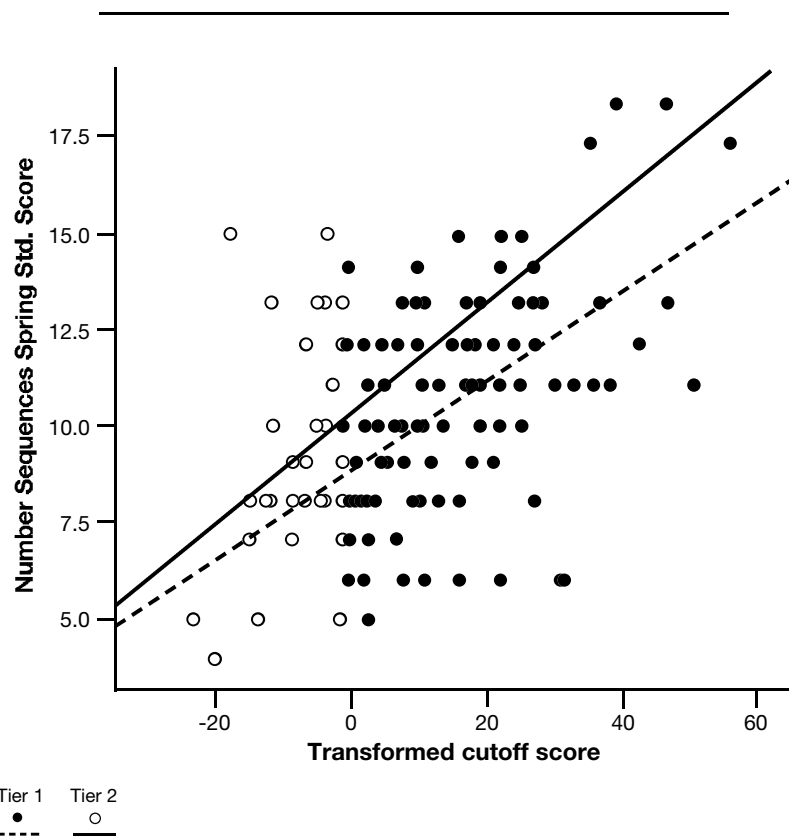
The RtI approach stipulates that evidence-based interventions must be implemented as part of a program to prevent learning difficulties and that progress monitoring results should help to inform decision-making regarding the learning disabilities identification process (D. Fuchs et al., 2003). Increasingly, the results of early mathematics intervention studies are helping to inform the field about what constitutes effective practices (e.g., Baker, Gersten, & Lee, 2002; L. Fuchs et al., 2005). Students with mathematics difficulties and those who are later identified as having learning disabilities in mathematics manifest problems in number sense, number and operation, and word problem solving (Bryant, Bryant, & Hammill, 2000; L. Fuchs

**Figure 1.** Positive main effect for spring TEMI standard score.



At 0 on x-axis, the regression line for Tier 2 is significantly above the line for Tier 1.

Figure 2. Positive main effect for spring number sequences standard score.



At 0 on x-axis, the regression line for Tier 2 is significantly above the line for Tier 1.

et al., 2003; Jordan et al., 2002). Thus, intervention programs that focus on these areas are critical.

The purpose of this study was to examine the effects of Tier 2 intervention booster lessons on number, operation, and quantitative reasoning performance of students in first grade who were identified as having mathematics difficulties. We extended our earlier research (see D. Bryant et al., 2008) by continuing to focus on number and operation skills and concepts related to number sense and arithmetic combinations but with increased duration and refinement in the booster lessons.

The TEMI-PM (2006-2007) total score, comprised of four subtests, was used to identify students who were struggling in critical early mathematics skills. The TEMI-

PM total score and subtest scores were also used to monitor student progress during Tier 2 intervention.

A total of 42 first-grade students participated in the Tier 2 intervention delivered as booster lessons. Twenty-minute intervention sessions were conducted four days per week for 23 weeks. Students received instruction with a high degree of fidelity in small groups by trained tutors. The intervention consisted of explicit, strategic instructional procedures, instructional content (e.g., numbers 0-99) for the booster lessons in number concepts, base-ten concepts, and addition and subtraction combinations with sums or minuends to 18, respectively.

Results from the RD analysis showed a significant main effect, indicating a positive program effect.

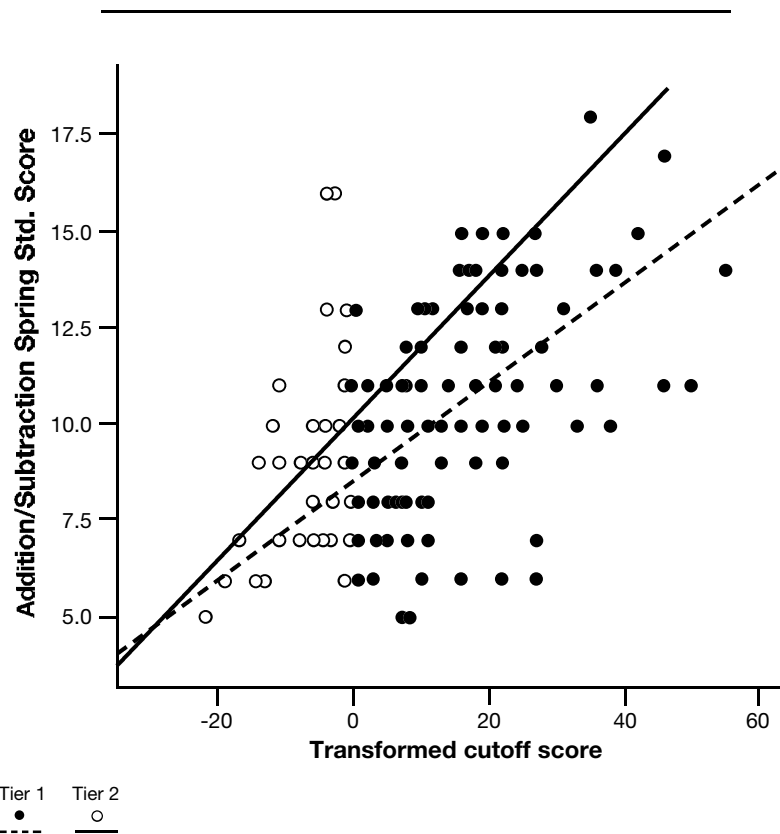
Findings are encouraging and support the notion that the number sense and arithmetic combinations performance of first-grade Tier 2 students can be improved with preventive intervention instruction. The additional day of instructional time coupled with the lengthened intervention (i.e., 23 weeks) may help explain the overall findings; these changes may have provided struggling first graders with the type of assistance they needed to improve performance. Also, lessons focusing on number sense tasks (e.g., magnitude, relationships of 10) and arithmetic combinations apparently provided the necessary “boost” students needed to strengthen many of the Tier 2 students’ ability.

Upon examination of subtest results to further explain the overall program effect, we were pleased to find a pro-

gram effect for number sequences and arithmetic combinations. The Number Sequences subtest involves several abilities, including number recognition, counting, identification of the missing number in the beginning (e.g., \_\_\_ 31 32), middle (e.g., 40 \_\_\_ 42), or end (e.g., 65 66 \_\_\_) position in a three-number sequence, and “code switching” across three-number sequence items (e.g., 1st item: 40 \_\_\_ 42; 2nd item: \_\_\_ 60, 61) fluently. As part of the booster lessons, students had multiple opportunities to use number cards activities and 100’s charts to learn and practice the skills associated with number sequences; apparently, this practice contributed to their improved performance on the TEMI-PM.

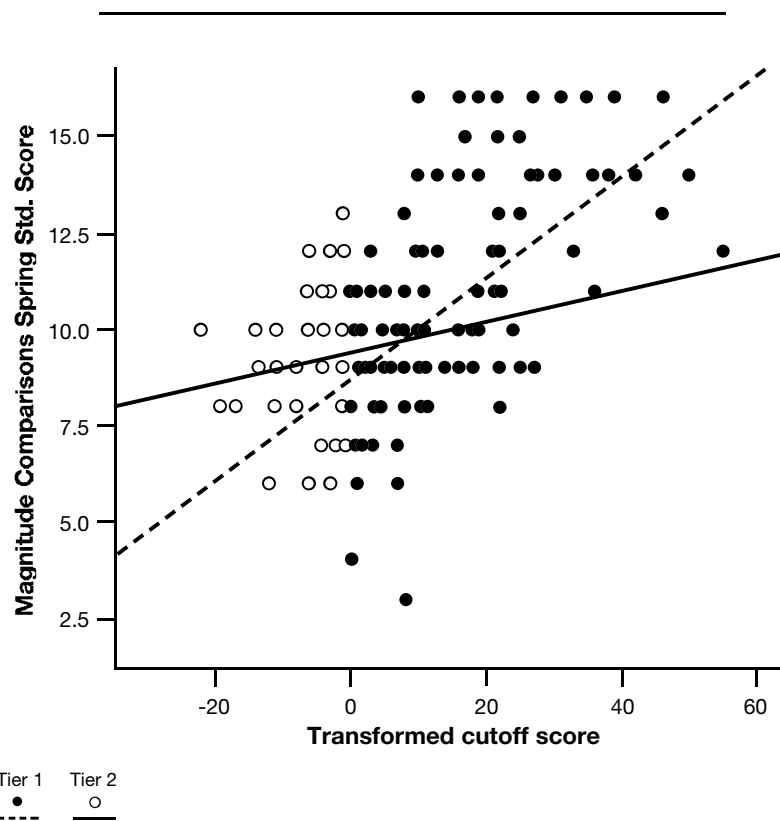
Also, obtaining a program effect for arithmetic combinations was encouraging. This TEMI-PM subtest,

**Figure 3.** Positive main effect for spring addition/subtraction standard score.



At 0 on x-axis, the regression line for Tier 2 is significantly above the line for Tier 1.

**Figure 4.** Interaction effect for spring magnitude comparison standard score.



At 0 on x-axis, the regression line for Tier 2 is very close to the line for Tier 1, however it is increasingly above the line for Tier 1 to the left of 0. This indicates that students with the lowest scores benefitted from Tier 2 intervention.

Addition/Subtraction Combinations, required that students compute addition and subtraction facts (mixed problems) in a 2-minute time period. Thus, students had to recall the answers to facts using whatever strategy worked for them and again “code switch” between types of problems (i.e., addition and subtraction). Improvement in this skill (i.e., fact families, and addition and subtraction of sums and minuends to 18, respectively) is often challenging to obtain with struggling students and takes time (D. Bryant et al., 2008; L. Fuchs et al., 2006). Conceivably, work with CSA representations, fact families (e.g.,  $5 + 6$ ,  $6 + 5$ ,  $11 - 6$ ,  $11 - 5$ ), and “fast facts” (i.e., fluency building activity) contributed to the improvement students were able to obtain with arithmetic combinations.

Finally, the interaction effect with the Magnitude Comparisons subtest showed more favorable results for students with the lowest scores. That is, the lower functioning Tier 2 students showed the most progress on this measure. Within a 2-minute timed period, this subtest requires students to recognize two numerals in an item and to discriminate between the quantities to decide which numeral represents the smaller amount or whether the quantities are the same (i.e., equal). We employed a CSA approach, which especially benefited the lower performing students and may have influenced learning to a greater degree than for higher Tier 2 students.

Another possible explanation for the positive program effects is that the two tutors who worked with the

---

first-grade students were “veteran” tutors and, thus, were more comfortable with the intervention and the participating school. The tutors knew the routine and established a materials management system that worked efficiently for them, a critical factor when providing briskly paced Tier 2 intervention. Additionally, both of these tutors demonstrated capable behavior management skills that limited off-task, disruptive student behavior. The inclusion of the behavior management system may also have contributed to the positive findings. Students seemed interested in working for their daily stickers, which were prominently displayed on the tutoring sticker poster in the tutors’ classroom. Positive behavior also generalized to praise and rewards by the classroom teacher as part of the school-wide behavior management plan.

However, more refinement of the booster lessons is needed to help close the achievement gap for all students. As illustrated when examining Figure 1 (total score for the TEMI-PM), there remains a group of students whose spring performance (i.e., standard score on the TEMI-PM) falls short of adequate response to intervention. For these students, we refer to Hallahan’s (2006) comment about what is necessary to provide more support; that is, the need for “intensive, relentless, iterative, individualized instruction.”

L. Fuchs and her colleagues (see their article in this issue) offer recommendations regarding principles of effective tertiary (Tier 3) intervention. We would add to their recommendations the need to conduct dynamic assessment to gain a better grasp of how this group of students understands (or misunderstands) mathematical concepts and what intensity of instruction is needed to elicit more adequate RtI. Thus, future studies should examine additional tutoring features to help Tier 2 students who have low levels (i.e., flat slope) of response and to develop more individualized interventions that can be considered Tier 3 (tertiary) intervention.

### **Limitations**

There are several limitations to this research. For example, a larger sample size in a study with an experimental design is warranted to further validate our findings. Also, research in word problem solving with struggling students in the primary grades requires further attention, although recent studies (e.g., L. Fuchs et al., 2005; L. Fuchs et al., 2007) are helping to inform the field about this area. Finally, the posttest measures (TEMI-PM) are aligned tightly with the content of Tier 2 intervention. However, the domains taught in Tier 2 instruction and measured on the TEMI-PM are suggested by the developmental psychology literature as crucial components for success in subsequent work in mathematics (Jordan et al., 2006). Thus, teaching early

numeracy skills and employing curriculum-based measures to assess these early foundation skills appears to be a viable approach to preventive intervention in the primary grades.

### **Implications of the Findings**

In terms of translating research to practice, several implications from this study should be considered. First, since Tier 2 intervention in mathematics is a critical component of instruction, schools must determine how to integrate secondary interventions into teachers’ daily schedules, which are already packed with core instruction in various subject areas, “specials” (e.g., music and art), and pullout programs (e.g., ESL, reading intervention).

In our current work, which includes working with schools to learn more about how educators tackle including multiple instructional agendas, we are seeing the challenges that educators are facing as they attempt to make RtI a reality. Time for Tier 2 intervention is the number one challenge, followed closely by what to do with the other students while secondary interventions are taking place with Tier 2 students by general education teachers. Findings from this study support the need to provide interventions that are at least 20 minutes long each session for four days a week across the majority of the school year. Creative master schedules developed in the spring for the following year should incorporate time for educators to provide the Tier 2 intervention in mathematics (as well as reading, which is often needed with students who have math difficulties) that struggling students desperately need.

Second, skills that focus on number and operation plus word problem solving are critical components of Tier 2 instruction. Tier 2 students can benefit from more focused teaching in number sense and arithmetic combinations. Third, explicit, systematic instruction produces positive change in student performance (Kroesbergen & Van Luit, 2003), especially for students with greater risk factors (e.g., low income, limited informal mathematics experiences before kindergarten) (Jordan, Kaplan, Locuniak, & Ramineni, 2007). Core mathematics instruction can benefit from the inclusion of more information for classroom teachers on how to use the critical features of instruction to teach students who are struggling (B. Bryant, Bryant, Kethley, Kim, Pool, & Seo, 2008). Such instruction should also include different types of representations (i.e., CSA) to facilitate conceptual understanding (Miller & Hudson, 2007).

In sum, Tier 2 intervention for students with mathematics difficulties holds promise for improving mathematics performance in number sense tasks and arithmetic combinations. Practitioners can implement

small-group intervention focusing on number, operation, and quantitative reasoning tasks with some assurance that a group of their struggling students will benefit. Finally, instruction that includes explicit, strategic procedures along with materials that engage students in representing numerical concepts and arithmetic combinations should be included as part of Tier 2 intervention.

## REFERENCES

- Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103, 51-73.
- Baker, S., Gersten, R., Flojo, J., Katz, R., Chard, D., & Clarke, B. (2002). *Preventing mathematics difficulties in young children: Focus on effective screening of early number sense delays* (Technical Report No. 0305). Eugene, OR: Pacific Institutes for Research.
- Bell, M., Bell, J., Bretzlaugh, J., Dillard, A., Hartfield, R., Isaacs, A., McBride, J., Pitvorec, K., & Saecker, P. (2001). *Everday mathematics: The University of Chicago school mathematics project*. Chicago: SRA/McGraw-Hill.
- Bryant, B. R., Bryant, D. P., Gersten, R., Wagner, R., Roberts, G., Kim, S., Seo, Y., & Shih, M. (2008). *Technical characteristics of early mathematics progress monitoring measures for kindergarten, first grade, and second grade*. Manuscript in preparation.
- Bryant, B. R., Bryant, D. P., Kethley, C., Kim, S. A., Pool, C., & Seo, Y. (2008). Preventing mathematics difficulties in the primary grades: The critical features of instruction in textbooks as part of the equation. *Learning Disability Quarterly*, 31, 21-35.
- Bryant, D. P., Bryant, B. R., Gersten, R., Scammacca, N., & Chavez, M. (2008). Mathematics intervention for first- and second-grade students with mathematics difficulties: The effects of Tier 2 intervention delivered as booster lessons. *Remedial & Special Education*, 29(1), 20-32.
- Bryant, D. P., Bryant, B. R., & Hammill, D. D. (2000). Characteristic behaviors of students with learning disabilities who have teacher-identified math weaknesses. *Journal of Learning Disabilities*, 33(2), 168-177, 199.
- Bryant, D. P., Smith, D. D., & Bryant, B. R. (2008). *Teaching students with special needs in inclusive classrooms*. Boston: Allyn & Bacon.
- Butler, F. M., Miller, S. P., Crehan, K., Babbit, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, 18, 99-111.
- California State Board of Education. (2007). *Mathematics content standards*. Sacramento: Author. Retrieved January 30, 2008, from <http://www.cde.ca.gov/be/st/ss/mthgrade1.asp>
- Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, 15(3), 179-202.
- Case, R., & Okamoto, Y. (1996). The role of conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development*, 61(12) (Serial No. 246).
- Chard, D. J., Baker, S. K., Clarke, B., Jungjohann, K., Davis, K., & Smolkowski, K. (2008). Preventing early mathematics difficulties: The feasibility of a rigorous kindergarten mathematics curriculum. *Learning Disability Quarterly*, 31(1), 11-20.
- Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment for Effective Intervention*, 30(2), 3-14.
- Charles, R. I., Chancellor, D., Moore, D., Schielack, J. F., Van de Walle, J., et al. (1999). *Math*. Menlo Park, CA: Scott Foresman-Addison Wesley.
- Clarke, B., & Shinn, M. R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review*, 33(2), 234-238.
- Fuchs, D., Mock, D., Morgan, P. L., & Young, C. L. (2003). Responsiveness-to-intervention: Definitions, evidence, and implications for the learning disabilities construct. *Learning Disabilities Research & Practice*, 18(3), 157-171.
- Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology*, 97, 493-513.
- Fuchs, L. S., & Fuchs, D. (1998). Treatment validity: A unifying concept for reconceptualizing the identification of learning disabilities. *Learning Disabilities Research & Practice*, 13, 204-219.
- Fuchs, L. S., & Fuchs, D. (2001). Principles for the prevention and intervention of mathematics difficulties. *Learning Disabilities Research & Practice*, 16, 85-95.
- Fuchs, L., Fuchs, D., Compton, D. L., Bryant, J. D., Hamlett, C. L., & Seethaler, P. M. (2007). Mathematics screening and progress monitoring at first grade: Implications for responsiveness to intervention. *Exceptional Children*, 73(3), 311-330.
- Fuchs, L. S., Fuchs, D., Hamlet, C. L., Powell, S. R., Capizzi, A. M., & Seethaler, P. M. (2006). The effects of computer-assisted instruction on number combination skill in at-risk first graders. *Journal of Learning Disabilities*, 39(5), 467-475.
- Fuchs, L. S., Fuchs, D., & Hollenbeck, K. N. (2007). Extending responsiveness to intervention to mathematics at first and third grades. *Learning Disabilities Research & Practice*, 22(1), 13-24.
- Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R., et al. (2003). Enhancing third-grade students' mathematical problem solving with self-regulated learning strategies. *Journal of Educational Psychology*, 95, 306-315.
- Fuchs, L., Fuchs, D., & Speece, D. L. (2002). Treatment validity as a unifying construct for identifying learning disabilities. *Learning Disability Quarterly*, 25, 33-46.
- Fuchs, L. S., Fuchs, D., Yazdian, L., & Powell, S. R. (2002). Enhancing first-grade children's mathematical development with peer-assisted learning strategies. *School Psychology Review*, 31, 569-583.
- Fuson, K. C. (1990). Issues in place-value and multidigit addition and subtraction learning and teaching. *Journal for Research in Mathematics Education*, 21(4), 273-280.
- Geary, D. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, 33, 386-404.
- Geary, D. C. (2003). Learning disabilities in arithmetic: Problem solving differences and cognitive deficits. In H. L. Swanson, K. Harris, & S. Graham (Eds.), *Handbook of learning disabilities* (pp. 199-212). New York: Guilford Publishers.
- Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37, 4-15.
- Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology*, 77, 236-263.
- Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
- Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education*, 33, 18-28.



- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293-304.
- Griffin, S. A. (2004). Building number sense with Number Worlds: A mathematics program for young children. *Early Childhood Research Quarterly, 19*, 173-180.
- Griffin, S. A., & Case, R. (1997). Re-thinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education, 3*, 1-49.
- Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25-33.
- Hallahan, D. P. (2006, April). *Challenges facing the field of learning disabilities*. Presentation at the National SEA Conference on SLD Determination, Kansas City, MO.
- Hanich, L., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*(3), 615-626.
- Harcourt Assessment. (2003). *Stanford achievement test-Tenth edition*. San Antonio, TX: Author.
- Heller, K. A., Holtzman, W. H., & Messick, S. (Eds.). (1982). *Placing children in special education: A strategy for equity*. Washington, DC: National Academy Press.
- Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics, 23*(2), 98-122.
- Individuals with Disabilities Education Improvement Act of 2004*. Pub. L. No. 108-446.
- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology, 85*, 103-119.
- Jordan, N. C., Kaplan, D., & Hanich, L. B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of Educational Psychology, 94*, 586-597.
- Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense: Trajectories. *Learning Disabilities Research & Practice, 22*(1), 36-46.
- Jordan, N. C., Kaplan, D., Oláh, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153-175.
- Kroesbergen, E. H., & Van Luit, J.E.H. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education, 24*, 97-115.
- Massachusetts Department of Education. (2000). *Massachusetts mathematics curriculum framework*. Boston: Author. Retrieved January 30, 2008, from <http://www.doe.mass.edu/frameworks/math/2000/final.pdf>
- Mercer, C. D., Jordan, L., & Miller, S. P. (1996). Constructivistic math instruction for diverse learners. *Learning Disabilities Research and Practice, 11*, 147-156.
- Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice, 22*(1), 47-57.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Okamoto, Y., & Case, R. (1996). Exploring the microstructure of children's central conceptual structures in the domain of number. *Monographs of the Society for Research in Child Development, 61*, 27-59.
- Ostad, S. (1998). Developmental differences in solving simple arithmetic word problems and simple number-fact problems: A comparison of mathematically disabled children. *Mathematical Cognition, 4*(1), 1-19.
- Ross, S. H. (1989). Parts, wholes, and place value: A developmental view. *Arithmetic Teacher, 36*(6), 47-51.
- Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston: Houghton-Mifflin.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*(2), 428-444.
- Texas Education Agency. (2006). *Texas essential knowledge and skills*. Austin: Author. Retrieved January 30, 2008, from <http://www.tea.state.tx.us/rules/tac/chapter111/ch111a.html>
- Trochim, W. (1984). *Research design for program evaluation: The regression-discontinuity approach*. Beverly Hills, CA: Sage.
- University of Texas System/Texas Education Agency. (2006). *Texas early mathematics inventories: Progress monitoring*. Austin: Author.
- VanDerHeyden, A. M., Witt, J. C., Naquin, G., & Noell, G. (2001). The reliability and validity of curriculum-based measurement readiness probes for kindergarten students. *School Psychology, 30*, 363-382.
- Van de Walle, J. A. (2004). *Elementary and middle school mathematics: Teaching developmentally*. Boston: Allyn & Bacon.
- Vaughn, S., & Fuchs, L. (2003). Redefining learning disabilities as inadequate response to instruction: The promise and potential problems. *Learning Disabilities Research & Practice, 18*(3), 137-146.
- Vaughn, S., Linan-Thompson, S., & Hickman-Davis, P. (2003). Response to treatment as a means of identifying students with reading/learning disabilities. *Exceptional Children, 69*, 391-409.

#### AUTHORS' NOTE:

The work on this article was supported in part by a grant # 076600157110004 from the Texas Education Agency. Statements do not support the position or policy of this agency, and no official endorsement should be inferred.

Please address correspondence to: Diane Pedrotty Bryant, College of Education, The University of Texas, Austin, TX 78712; [BrianRBryant@aol.com](mailto:BrianRBryant@aol.com)