

Fraction Interventions for Students Struggling to Learn Mathematics: A Research Synthesis

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Abstract

This study synthesized intervention studies focusing on instruction to improve fraction skills. Seventeen studies met the inclusion criteria: being published in English-language peer-reviewed journals or dissertations between 1975 and 2014, and targeting 3rd- through 12th-grade students struggling to learn mathematics. From the *Common Core State Standards for Mathematics*, addition and subtraction of fractions were most frequently representing the *Standards for Mathematical Content*, and modeling for mathematics instruction was most frequently observed to represent the *Standards for Mathematical Practice*. Results indicated that interventions consisting of evidence-based instructional components (e.g., concrete and visual representations; explicit, systematic instruction; range and sequence of examples; heuristic strategies; and use of real-world problems) led to improved performance on measures with fraction concepts and skills. Limitations and directions for future research are discussed.

Keywords

fraction instruction, low-achieving students, mathematics difficulties, mathematics learning disabilities, Common Core State Standards for Mathematics

Success in algebra is considered to be the “gatekeeper” to postsecondary education and essential for many careers (National Mathematics Advisory Panel [NMAP], 2008). Mastering critical algebraic concepts, such as proportional reasoning and fractions, facilitates the learning of more advanced mathematical ideas. Many postsecondary degree programs require mastery of algebra content (Ketterlin-Geller & Chard, 2011).

In recent years, nearly all states have established more rigorous mathematics requirements, including successful completion of Algebra 1, for high school graduation (American Diploma Project, 2004). Notably, students must learn mathematical topics, including fractions, prior to algebra instruction to be able to tackle the rigorous demands associated with this content area.

Unfortunately, the National Assessment of Educational Progress (National Center for Education Statistics, 2013) indicated that about half of students in 8th and 12th grade lack the conceptual understanding and procedural knowledge that is critical for competence with fractions. Among fourth graders with and without disabilities, only about 31% could predict the first fraction in a pattern that was greater than 1, and only about 35% could solve a problem using operations with fractions. Among eighth graders, about 41% could solve a multistep problem involving fractions (National Center for Education Statistics, 2013). By

comparison, according to the 2011 Trends in International Mathematics and Science Study, both fourth and eighth graders in East Asia countries outperformed those in the United States on mathematical reasoning, including fractions understanding and knowledge (Mullis, Martin, Foy, & Arora, 2012). Thus, for U.S. students, performance outcomes for fractions, including for students with academic difficulties, are alarming, given the importance of competence with fractions as part of the learning progression for algebra.

As one of the critical foundations of algebra, conceptual understanding of fractions is considered an essential building block for successfully advancing in elementary and secondary mathematics. Conceptual understanding is defined as “implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain” (pp. 346–347), whereas procedural understanding refers to “the ability to execute action sequences to solve problems” (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346).

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Unfortunately, understanding the various concepts for fractions is challenging for all students, including students struggling to learn mathematics; hence, fractions is an area in which students need additional support and instruction (Dougherty, Bryant, Bryant, Darrough, & Pfannenstiel, 2014; NMAP, 2008). Specifically, these students demonstrate difficulties with rank-ordering fractions and identifying equivalent fractions (Grobeck, 2000; Hecht & Vagi, 2010; Mazzocco & Devlin, 2008), signifying a lack of conceptual understanding of fractions that can lead to misconceptions with fractions that impinge on the successful performance of fractions. For example, students incorrectly name fractions when equal parts are not shown in the figure (e.g., rectangular), failing to visualize equally sized parts in the whole (Barnett-Clarke, Ramirez, & Coggins, 2010). Also, lack of conceptual understanding of fractions can, in turn, limit students' ability to apply routine computational procedures involving fractions (Siegler et al., 2010).

Such lack of conceptual understanding of and computational facility with fractions limits students' ability to solve more advanced computational problems, including ratios, rates, and proportions, all of which are critical foundational skills for algebra (Siegler et al., 2010). To remedy this situation and thereby better prepare students for success in the 21st century, teachers must be able to access interventions that focus on critical concepts and procedures associated with teaching fractions to all students, including those who find mathematics instruction difficult.

A Framework for Fraction Instruction

For more than a decade, leading professional groups have identified important conceptual understanding and procedural knowledge for fraction instruction that students must master in preparation for algebra and framed mathematics instruction and provided guidance for fraction instruction, in particular. For example, the National Council of Teachers of Mathematics (NCTM; 2000), in *Principles and Standards for School Mathematics*, offered recommendations for instruction on fractions, in its *Content Standards*. Furthermore, the NCTM (2006) published the *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics*, which emphasized an understanding of fractions concepts and skills in Grades 3 through 6.

More recently, the *Common Core State Standards for Mathematics* (CCSSM; National Governors Association Center for Best Practices [NGAC] & Council of Chief State School Officers [CCSSO], 2010), professional organizations, and policy makers have emphasized the importance of developing conceptual understanding of fractions and computational procedural knowledge. Promoting this understanding is formalized beginning in Grade 3 through the modeling (showing) of fraction concepts. In Grade 5,

knowledge of whole-number multiplication and division is then applied to multiplying and dividing fractions.

In addition, professionals have been emphasizing the importance of mathematical practices in developing conceptual understanding of fractions. The NCTM's (2000) Process Standards provide a solid framework for developing conceptual understanding. More recently, and similarly, the CCSSM (NGAC & CCSSO, 2010) put forth *Standards for Mathematical Practices*, which focused on problem solving and modeling mathematics, for example. The NMAP (2008) also signaled the significance of mathematical reasoning and problem solving in teaching fraction concepts and operational procedures and how to connect fraction skills to algebraic concepts. Thus, both content and practices for fraction instruction are well framed by contributions from the NCTM (2000, 2006), NGAC and CCSSO (2010), and NMAP (2008). Equally important are evidence-based instructional components (e.g., using visual representations) for teaching fractions to students who struggle with mathematical understanding of fractions, as presented below.

Key Evidence-Based Instructional Components in Teaching Fractions

Despite the increased grade-level expectations for mathematical content and mathematical practices for fraction instruction, a high percentage of students still lack conceptual and procedural understanding of fractions and, therefore, entertain certain misconceptions about fractions. As a result, professionals have been focusing on identifying effective instructional components to prevent and remediate students' misconceptions about fractions. To that end, they have emphasized several instructional components as important for designing effective fraction interventions for struggling learners. For instance, one meta-analysis (Gersten et al., 2009) on mathematics instruction for students with learning disabilities (LD) found that explicit instruction, use of heuristic strategies, and use of real-world problems were effective instructional components for the design of curriculum and instruction.

More recently, the Institute of Education Sciences practice guide on effective fraction instruction (Siegler et al., 2010) highlighted the use of representations (e.g., number lines) and the development of both a procedural and conceptual understanding of fractions. For instance, to help students understand and connect the mathematical ideas associated with the procedure of multiplying fractions, modeling or representing this procedure has been found to enhance a more abstract conceptual understanding of the operation.

A series of grade-level publications by the NCTM (e.g., Focus in Grade 3 through Focus in Grade 6) has also endorsed the use of visual representations for teaching

fractions. Specifically, research suggests that mathematical tasks include number lines, area models, and fraction-bar models to represent fractions, along with money, clocks, and place value. The connection of visual representations and fractions is especially important in developing a conceptual understanding of fractions and helps students engage in the mathematical reasoning of problem solving (Empson & Levi, 2011).

Previous Syntheses on Fraction Interventions

As a group, students struggling to learn mathematics demonstrate lower performance on fraction items on national assessments than their typically achieving peers (National Center for Education Statistics, 2013). Thus, there is a need to better understand the research related to fraction interventions for this group of students.

However, despite the growing emphasis on fraction instruction for students struggling to learn mathematics, previous syntheses on effective mathematics instruction have focused on general mathematics areas (e.g., Gersten et al., 2009) or specifically addressed word problems (WP) or algebra (e.g., Powell, 2011). Only one study (Misquitta, 2011) reviewed the qualities and effectiveness of fraction instruction for struggling mathematics learners from English-language peer-reviewed journals published between 1990 and 2008. In the current study, we expanded Misquitta's (2011) review by extending the inclusion years from 1975 to 2014, connecting the CCSSM (NGAC & CCSSO, 2010) to each article's topics, and analyzing the presence of the instructional components embedded in fraction instruction. These key differences extend the findings from Misquitta's review and contribute to the importance of the current synthesis.

The purpose of this literature synthesis was to identify intervention studies that focused on fraction interventions for students with mathematics learning difficulties. Particularly, this synthesis provides insights into the features and instructional components of targeted intervention studies to determine the nature of fraction instruction for students who struggle with learning mathematics. Moreover, this synthesis sought to determine the extent of research on fractions as it relates to the CCSSM (NGAC & CCSSO, 2010) because of the national attention devoted to a more focused curriculum. Finally, we calculated effect sizes (ESs) to determine the effect of the interventions on performance in fractions.

The research questions were as follows:

Research Question 1: What are the features of fraction intervention studies (e.g., participants) for teaching fractions to students struggling to learn mathematics?

Research Question 2: Which CCSSM are represented in each fraction intervention study?

Research Question 3: What are the instructional components (e.g., explicit, systematic instruction) of the fraction interventions?

Research Question 4: How effective are the fraction interventions in improving the mathematics performance of students struggling to learn mathematics?

Method

Literature Search Procedures

We reviewed all studies published between 1975 and 2014 that focused on interventions to improve the fraction performance of students struggling to learn mathematics. The year 1975 was selected as the starting point because it marked the earliest definition found for students with LD in the Individuals With Disabilities Education Act (2004).

The search consisted of the following three steps. First, an online search was performed, using electronic databases such as ERIC, JSTOR, PsychInfo, and ProQuest Dissertations & Theses Full Text. A list of key words (e.g., *fraction*, *instruction*, *low achieving*, *learning disabilities*) and their combinations was entered to identify potential studies in the literature. Next, a manual search of journals was carried out of the following publications: *Exceptional Children*, *Intervention in School and Clinic*, *Journal of Learning Disabilities*, *The Journal of Special Education*, *Learning Disability Quarterly*, *Learning Disabilities Research & Practice*, and *Remedial and Special Education*. Last, the references of research literature reviews on teaching students struggling to learn mathematics were examined (e.g., Gersten et al., 2009). This search resulted in 459 studies. After reviewing the titles, key words, and abstracts of these studies, 40 studies were selected for further review. Of the 40 studies, 17 (43%) met the following inclusion criteria for in-depth analysis:

- a. Participants were students struggling to learn mathematics (see the operational definition below). In the case of studies that included students struggling to learn mathematics as part of an overall group of participants, to be included, the data for the target participants had to be disaggregated, or at least 50% of the study participants had to be students struggling to learn mathematics.
- b. Participants were in 3rd through 12th grades.
- c. Studies were conducted in the United States and published in English-language peer-reviewed journals or dissertations between 1975 and 2014. In an effort to distinguish this synthesis from Misquitta's (2011), similar to Ritchey and Goeke (2006), we included dissertations to broaden the review and research findings.

- d. Studies included an intervention designed to teach fractions as the independent variable.
- e. Studies included outcomes of fraction instruction as the dependent variable. Studies that included fraction instruction as only a part of the intervention had to separately measure performance in fractions to be included.
- f. Studies used randomized controlled trials, quasi-experimental designs, or single-subject designs.
- g. Studies stated the number of participants and statistical information (e.g., means, standard deviations, *t*-test values, *f*-test values) needed to calculate ESs.

To ensure that studies were selected using the same criteria, another researcher randomly selected 5 of 17 studies (29%). Inter-rater reliabilities of whether to include each study was 93%; this was calculated by taking the number of agreements and dividing by the number of agreements plus disagreements multiplied by 100. After discussion, raters achieved 100% agreement on the inclusion of studies.

Of the 17 studies included in this synthesis, 12 used a randomized controlled trial design, 1 applied a quasi-experimental design, and 4 implemented a single-subject research design. The remaining 23 studies did not meet the criteria and were, therefore, excluded for the following reasons: 12 did not include disaggregated data for fractions, 4 implemented single-group pretest–posttest designs, 4 were conducted outside the United States, and the remaining 3 did not provide enough information to calculate ESs.

In comparison with Misquitta's (2011) review, of the 17 studies qualifying for the present synthesis, 10 studies (Bottge et al., 2014; Bottge, Rueda, Grant, Stephens, & LaRoque, 2010; Courey, 2006; Hughes, 2011; Hunt, 2014; Kelly, Carnine, Gersten, & Grossen, 1986; Lambert, 1996; Reneau, 2012; Shin, 2013; Watt, 2013) were not included in Misquitta's review; of these, 6 dissertations were located (there were no dissertations in Misquitta's review), and 7 of the 10 studies were published more recently.

Data Analysis

The operational definition of *students struggling to learn mathematics* is described below. For the data analysis, we reviewed and coded the 17 studies that met the inclusion criteria and then calculated the effects of fraction interventions in each study.

Definition of students struggling to learn mathematics. Participants in the study were students struggling to learn mathematics. Researchers who conducted the studies in this synthesis used various criteria to demonstrate difficulties in mathematics. Thus, we operationally defined the term *students struggling to learn mathematics* as low-achieving students and students with LD. Low-achieving students were

defined as students who experienced difficulties in learning mathematics but were not identified as having LD and placed in remedial mathematics classes or scored at or below the 25th percentile based on grade-level norms. In turn, students with LD were defined as students whose school district had identified them as having LD.

Coding of studies. Each study was coded based on its features (e.g., participants were coded as total number of students, grade, disability status, ethnicity, and free or reduced-price lunch; measures as researcher-developed vs. standardized tests; instructors as teachers vs. researchers). Studies were also coded on the following from the Number and Operations–Fractions (NF) domain of the *CCSSM Standards for Mathematical Content* (e.g., comparing fractions; grade 3.NF domain.topic A.cluster 3 [3.NF.A.3]) and *Standards for Mathematical Practice* (MP; e.g., model with mathematics; MP 4; NGAC & CCSSO, 2010).

In addition, each study's instructional components were reviewed and organized into five categories: concrete and visual representations; explicit, systematic instruction; range and sequence of examples; heuristic strategies; and use of real-world problems. These categories were based on recommendations culled from meta-syntheses, syntheses, and reviews on teaching mathematics to students struggling to learn mathematics (Gersten et al., 2009; NMAP, 2008). The operational definitions of instructional components included the following (Gersten et al., 2009; NMAP, 2008):

- a. Concrete and visual representations indicated teachers using manipulatives, such as beans and construction paper, and visual representations, such as pictures and number lines, to demonstrate how to solve problems.
- b. Explicit, systematic instruction indicated that teachers demonstrated step-by-step strategies for solving problems, provided students with extensive opportunities to practice (e.g., guided practice), and gave corrective feedback and cumulative review.
- c. Range and sequence of examples indicated a specified sequence or pattern of examples (e.g., concrete to abstract) or variation in the range of examples.
- d. Heuristic strategies indicated cognitive strategies that anchor students to solve problems. The cognitive and meta-cognitive process of learning with self-regulatory strategies includes self-questioning skills for solving problems.
- e. Use of real-world problems indicated teachers applying contextualized problems to teach mathematical problem solving (e.g., anchored instruction).

Studies that included more than one instructional component were coded into more than one category whenever applicable. After the training, two trained coders

independently coded 24% ($n = 4$) of the 17 studies that met the inclusion criteria. For each category of features of studies, CCSSM, and instructional components, they coded each item *present* (1) or *absent* (0) and provided additional descriptions when the item was present. Inter-rater reliabilities were calculated using the following formula: Number of agreements / Number of agreements and disagreements \times 100%. Percentage of agreement was 96 for features of studies, 97 for CCSSM, and 100 for instructional components. After discussing items that were ambiguous, the 2 coders for each study achieved 100% agreement. Prior to the inter-rater reliability check, the two coders were trained to code each variable; they discussed the meaning of each category and practiced coding the data.

Calculation of ES. To examine the magnitude of effectiveness in each study, ES was calculated using Hedges' g for group studies. First, for randomized controlled trial studies, the effect size was calculated by computing the difference between the mean outcome of the treatment group and the mean outcome of the control group divided by the pooled within-group standard deviation (Cooper, Hedges, & Valentine, 2009). For quasi-experimental design studies, effect size was calculated by adjusting a pretest to remove bias resulting from the non-equivalence of participants (Wortman & Bryant, 1985). When a study reported t -test value (t) or F -test value (F) without reporting mean scores or standard deviations, additional formulas were applied: $t\sqrt{n_1 + n_2} / \sqrt{n_1 n_2}$ or $t\sqrt{F(m_1 + n_2)} / \sqrt{n_1 n_2}$ (n = number of participants). Finally, the estimate was corrected for small-sample bias using Hedges' g correction (Cooper et al., 2009; Hedges, 1981).

Next, in examining the effects of single-subject studies, the percentage of all non-overlapping data (PAND; Parker, Hagan-Burke, & Vannest, 2007), which provides an estimate of which data from the treatment phase improved over data from the baseline, was calculated (Maggin, Briesch, & Chafouleas, 2013). In addition, Tau-U, a non-parametric statistical measure of ES, which combines non-overlap data between phases with trend from within the intervention phase of all pair groups (Parker, Vannest, Davis, & Sauber, 2011), was calculated. Tau-U values were determined using $CI_{.90}$ and $CI_{.95}$ (CI = confidence interval). Tau-U was obtained using the online calculator from www.singlecaseresearch.org. According to Cohen's (1988) criterion, ESs around .80 are considered large, those around .50 are medium, and those below .20 are small. According to Scruggs and Mastropieri's (1994) recommendation, a percentage of non-overlapping data of 90 was considered very effective.

Results

For the 17 studies included in the synthesis, the features of fraction interventions and the CCSSM (NGAC & CCSSO,

2010) were reviewed. More important, the instructional components and the effects of fraction interventions represented in each study were also reviewed.

Features of Fraction Interventions

Participants and settings. A total of 805 students participated across the 17 studies. Of these students, 153 were low-achieving students, 301 were students with LD, 321 had other disabilities, and 30 were typically achieving students. Specifically, of the 301 students with LD, 175 had mathematics goals on their Individualized Education Programs (IEPs) indicating that they needed special education assistance in mathematics. Two studies included elementary school students (89 third graders). The remaining 15 studies included middle or high school students: 5 students in Grade 5, 32 in Grade 6, 8 in Grade 7, 26 in Grade 8, 36 in Grade 9, 477 in Grades 6 to 8, 56 in Grades 9 to 11, and 76 in Grades 9 to 12.

Furthermore, participants included 346 students classified as European American, 130 as African American, 25 as Hispanic, 8 as Native American, 2 as Asian American, 1 as Middle Eastern, and 6 as Other ethnicity; 52 were classified as unspecified, and 235 were not identified. Only six studies explicitly reported participants' socioeconomic status; of these, a majority of participants qualified for free or reduced-price lunch. Eight studies took place in a resource room, three in a remedial mathematics classroom, two in a general mathematics classroom, two in both a general mathematics classroom and technology education classroom or remedial mathematics classroom, and one in both a general mathematics classroom and a resource room; one study did not explicitly report the setting. Fifteen studies provided geographic locations: Southeast ($n = 7$), Midwest ($n = 3$), Northwest ($n = 2$), Southwest ($n = 2$), and South-central ($n = 1$).

Measures. Most studies ($n = 16$) implemented researcher-developed measures; four (Bottge et al., 2014; Bottge et al., 2010; Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Hunt, 2014) applied both standardized and researcher-developed measures. Thirteen studies reported internal consistency reliability (coefficient α ranging from .78 to .98) of the measures used in the research; only eight studies (Bottge, 1999; Bottge et al., 2014; Butler et al., 2003; Courey, 2006; Hunt, 2014; Lambert, 1996; Reneau, 2012; Shin, 2013) reported the validity of measures (e.g., criterion-related, construct validity, or face validity). Twelve studies included inter-rater reliability, ranging from 94% to 100%.

Fidelity of implementation. Most studies ($n = 16$) assessed the fidelity of interventions. One (Lambert, 1996) did not report fidelity of implementation. Nine included a procedural treatment fidelity check, reporting whether interventionists followed implementation procedures for main activities by

using lesson manuals or worksheets for guided and independent practice. In addition, seven other studies (Bottge et al., 2014; Butler et al., 2003; Hughes, 2011; Hunt, 2014; Kelly et al., 1986; Kelly, Gersten, & Carnine, 1990; Watt, 2013) checked both the procedures and the quality of treatment fidelity—that is, they observed whether interventionists controlled the instructional pace, organized materials, reinforced students' trials, provided feedback, and used terminology consistent with the targeted curriculum.

Instructor. In two studies (Bottge & Hasselbring, 1993; Kelly et al., 1990), teachers and researchers implemented the intervention together. Five other studies (Courey, 2006; Hunt, 2014; Reneau, 2012; Shin, 2013; Watt, 2013) included only researchers. Furthermore, of the 10 other studies with teacher-implemented lessons, 1 (Kelly et al., 1986) included general education or remedial mathematics teachers; 6 (Bottge et al., 2014; Bottge et al., 2010; Butler et al., 2003; Joseph & Hunter, 2001; Lambert, 1996; Test & Ellis, 2005) included special education teachers; 2 (Bottge, 1999; Bottge, Heinrichs, Mehta, & Hung, 2002) included general education mathematics, special education, and technology education teachers; and 1 (Watt, 2013) included general education mathematics and special education teachers.

Length of instruction. The length of instruction was determined by the number of sessions and the total amount of time devoted to instruction. Sessions were noted in terms of the number of lessons presented. Across the 17 studies, the number of sessions varied as follows: 6 to 10 sessions ($n = 10$), 12 to 21 sessions ($n = 4$), 24 to 30 sessions ($n = 2$), and 94 sessions ($n = 1$). Total amount of time devoted to instruction ranged from 180 to 5,640 min across the 14 studies: 180 to 450 min ($n = 9$), 450 to 540 min ($n = 1$), 600 min ($n = 2$), 1,200 to 1,320 min ($n = 1$), and 4,230 to 5,640 min ($n = 1$). Three studies (Bottge, 1999; Bottge et al., 2002; Joseph & Hunter, 2001) did not report the total amount of instructional time.

Instructional grouping. Three studies implemented only whole-group (Lambert, 1996), small-group (Joseph & Hunter, 2001), or pair (Test & Ellis, 2005) practices. The other 14 used mixed grouping, including whole-group, small-group, and independent practice (e.g., worksheet activities) provided at the end of each session. A summary of the key study characteristics is reported in Table 1.

CCSSM

The frequency analysis of the coded CCSSM (NGAC & CCSSO, 2010) focused on the Standards for Mathematical Content and Mathematical Practice in Number and Operations—Fractions domain. With regard to the *Standards for Mathematical Content*, 11 of the 17 studies focused on word problem solving with fractions. Of these 11 studies, 6

(Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2002; Bottge et al., 2014; Bottge et al., 2010; Reneau, 2012) included solving word problems involving addition and subtraction of fractions with like denominators (CCSSM 4.NF.B.3) and unlike denominators (CCSSM 5.NF.A.1), and 2 (Bottge et al., 2014; Bottge et al., 2010) of the 6 extended the concept of equivalent fractions (CCSSM 4.NF.A.1); 3 included solving word problems using multiplication of a fraction by a whole number (CCSSM 4.NF.B.4; Butler et al., 2003; Courey, 2006) and a fraction or whole number by a fraction (CCSSM 5.NF.B.4; Shin, 2013); 1 (Lambert, 1996) included multiplication of a fraction as well as addition and subtraction; and 1 (Hunt, 2014) focused on the concept of fraction equivalence (CCSSM 3.NF.A.3).

Six other studies focused on computation with fractions. Two studies (Joseph & Hunter, 2001; Test & Ellis, 2005) focused on adding and subtracting fractions with like and unlike denominators. Other four targeted at least two different standards. For example, three studies (Kelly et al., 1986; Kelly et al., 1990; Watt, 2013) focused on multiplication of a fraction (CCSSM 5.NF.B.4) or equivalent fractions (CCSSM 4.NF.A.2) as well as addition and subtraction with like denominators. Beyond the concept of multiplication of a fraction, the other study (Hughes, 2011) covered division of fractions and comparing fractions with different numerators and denominators.

Also, studies focused on four different *Standards for Mathematical Practice*. Most frequently used was modeling with mathematics (CCSSM MP 4). Fifteen of the 17 studies implemented visual representations by constructing tables and using diagrams, showing spreadsheet displays, or using numerical symbols (e.g., $_ + _ = []$) to translate a fraction picture to a numerical equation. The second most common practice was using appropriate tools (CCSSM MP 5); 12 studies used tools such as concrete materials (e.g., fractions circles, beans, and construction paper), virtual manipulatives, and videos. Twelve studies helped students to make sense of mathematical problem solving (CCSSM MP 1) by focusing on unknown solution methods via contextualizing problems or applying visual representations. Last, eight studies emphasized mathematical analytic reasoning with others (CCSSM MP 3) by having students express mathematical thinking through discourse (discussions and explanations) with peers or teachers.

Instructional Components and Effects of Interventions

The instructional components—including concrete and visual representations; explicit, systematic instruction; range and sequence of examples; heuristic strategies; and use of contextualized problems—were analyzed in all 17 studies. In addition, the effects of fraction interventions consisting of these instructional components were described.

Table 1. Descriptions of Study Characteristics.

Study	Participants	Sessions	Experimental conditions	Dependent measures	Effect size ^a		
Bottge (1999)	11 LA, 3 LD, 3 other 8th	10/Not reported	T: AI vs. C: WP	Computation Word problem Contextualized	-0.46 0.26 1.67		
Bottge, Heinrichs, Mehta, and Hung (2002)	7 LD, 1 other 7th	12/Not reported	T: EAI vs. C: WP	Computation Word problem Contextualized	-0.89 -0.14 0.92		
Bottge, Rueda, Grant, Stephens, and LaRoque (2010)	54 LD 6th–8th	24/50–55 min	T ₁ : Computer-based + EAI vs. T ₂ : EAI	Computation Problem solving ITBS computation ITBS problem solving	1.04 0.10 0.26 0.11		
Bottge et al. (2014)	49 LD, 286 other 6th–8th	94/45–60 min	T: EAI vs. C: Business as usual	Computation Problem solving ITBS computation ITBS problem solving	1.19 0.48 0.59 0.10		
Bottge and Hasselbring (1993)	19 LA, 17 other 9th	5/40 min	T: AI vs. C: WP	Word problem Contextualized	0.39 1.11		
Butler, Miller, Crehan, Babbitt, and Pierce (2003)	42 LD, 8 other 6th–8th	10/45 min	T ₁ : CRA vs. T ₂ : RA	Brigance area Brigance quantity Brigance abstract Improper Word problem	0.13 1.02 0.32 0.31 0.16		
Courey (2006)	48 LA, 1 LD, 2 other 3rd	6/30–40 min or 41–55 min	T ₁ : Procedural + conceptual supplement T ₂ : Procedural C: Basal	Half story Informal Conceptual Knowledge	T ₁ vs. C 1.70 0.70 0.22	T ₂ vs. C 1.88 -0.25 0.19	T ₁ vs. T ₂ 0.09 1.05 0.09
Hughes (2011)	14 LA, 20 LD, 1 other 6th–8th	30/20 min	T: CRA vs. C: Traditional	Computation	2.40		
Hunt (2014)	19 LA, 19 typical	20/30 min	T: Supplemental vs. C: Core instruction	Brigance Fraction equivalency	1.68 2.45		
Joseph and Hunter (2001)	3 LD 8th	15–21/Not reported	Cue card strategy	Teacher-made probes	PAND ^b 97%	Tau-U ^c 0.94	
Kelly, Carnine, Gersten, and Grossen (1986)	17 LD, 11 typical 9th–11th	10/30 min	T: VI vs. C: Basal	Criterion-referenced	1.01 1.16		
Kelly, Gersten, and Carnine (1990)	11 LA, 17 LD 9th–11th	10/45 min	T: VI vs. C: Basal	Curriculum-referenced	0.61		
Lambert (1996)	76 LD 9th–12th	8/55 min	T: Cognitive strategy vs. C: Textbook	Fractions			
Reneau (2012)	4 LA, 1 LD 5th	7–10/30 min	CRA	Probe	PAND ^b 91%	Tau-U ^c 0.83	
Shin (2013)	3 LD 6th–8th	9–10/30 min	Fun fraction	Probe	PAND ^b 82%	Tau-U ^c 0.74	
Test and Ellis (2005)	3 LD, 3 other 8th	15–18/30 min	LAP instruction	LAP fraction test	PAND ^b 100%	Tau-U ^c 1.00	
Watt (2013)	27 LA, 5 LD 6th	10/30 min	T: CRA vs. C: Traditional	Fraction unit	1.05		

Note. Other = other disabilities; LA = low achieving; LD = learning disabilities; C = comparison or control condition; T, T₁, T₂ = treatment condition; AI = anchored instruction; WP = word problem; EAI = enhanced anchored instruction; ITBS = *Iowa Tests of Basic Skills*; CRA = concrete-representational-abstract; RA = representational-abstract; PAND = percentage of all non-overlapping data.

^aEach effect size is calculated for each dependent measure. ^bPAND represents the average across all students. ^cTau-U represents the weighted average across all students.

Concrete and visual representations and range and sequence of examples. The most commonly used instructional component was concrete and visual representations. In one study (Butler et al., 2003), while applying concrete–representational–abstract (CRA) instruction, teachers used concrete and visual representations in conjunction with sequence of examples. Specifically, in introducing fraction concepts and skills, teachers used concrete objects such as fraction strips and folded construction paper to aid in students’ understanding of equivalent fractions, beans to represent the denominator of equivalent fractions, and other commercial fraction circles and representational drawings in a sequential manner. Students also learned how to use those manipulatives and drawings while solving fraction problems.

In Butler et al. (2003), the treatment group primarily comprised students with LD receiving CRA instruction in general education mathematics classes. After 10 lessons (45 min per session), those students performed better than the group with only representational–abstract (RA) instruction on all measures and performed significantly better on the Quantity Fractions subtest of the Brigance assessment ($ES = 1.02$, $p < .0005$), demonstrating a significant increase in conceptual understanding of fraction equivalency. The effects of CRA instruction on other subtests of the Brigance, including Area Fractions and Abstract Fractions, and on the researcher-developed measures of Word Problems and Improper Fractions ranged between small and moderate ($ES = 0.13$ – $.32$).

Concrete and visual representations and explicit, systematic instruction. Two studies (Courey, 2006; Hunt, 2014) used visual representations via explicit, systematic instruction.

In Courey (2006), researchers taught “half” word problems by implementing procedural instruction through visual representations (e.g., circles) and providing additional conceptual supplements, using conceptually laden language. Procedural instruction was more effective in improving students’ problem solving with fractions ($ES = 1.88$) than in improving their conceptual ($ES = 0.19$) or informal knowledge ($ES = -0.25$). On informal knowledge tasks, the group receiving the combined treatment of procedural instruction and conceptual supplement outperformed the group receiving only procedural instruction ($ES = 1.05$).

Hunt (2014) examined the effects of Tier 2 instructional intervention on fraction equivalency concepts and applications. The supplemental intervention consisted of explicit and conceptual instructional components, including cumulative reviews, systematic lesson sequence, and concrete and visual representations. Nineteen third graders receiving core instruction plus intervention outperformed those receiving only core instruction on Brigance assessment

($ES = 1.68$) as well as a researcher-developed fraction equivalency test ($ES = 2.45$).

Concrete and visual representations; explicit, systematic instruction; and range and sequence of examples. Four studies (Hughes, 2011; Kelly et al., 1986; Kelly et al., 1990; Watt, 2013) implementing concrete and visual representations did so with two other instructional components: explicit, systematic instruction and range and sequence of examples. First, in two group studies (Kelly et al., 1986; Kelly et al., 1990), interventionists provided systematic instruction while presenting secondary students with fraction pictures embedded in videodisc software. For example, students first translated fraction pictures into numerical equations. Teachers introduced the procedural rule of adding fractions after students had developed a conceptual understanding of adding fractions through fraction pictures. Moreover, lessons addressed the differences between addition and multiplication and, after students had practiced enough to be able to discriminate between these two problem types, their skills were integrated with other types of fraction problems; fraction terms such as *numerator* and *denominator* were presented separately to prevent student confusion over the terminology (Kelly et al., 1986; Kelly et al., 1990).

In the two studies (Kelly et al., 1986; Kelly et al., 1990), teachers also presented a wide range of examples by teaching how to read and write both proper and improper fractions from the beginning to avoid misconceptions (i.e., incorrect ideas about a concept) about fractions and by providing fraction problems with the unknown value on both the left and right sides of the equal sign (Kelly et al., 1990). The range of examples was distinct from instruction for the control group. In the control group (e.g., basal instruction), teachers presented only proper fractions (e.g., fractions less than 1), and the unknown value was always presented on the right side of the equal sign (Kelly et al., 1990). Thus, secondary students who received videodisc instruction (VI) significantly outperformed students receiving the basal (textbook) curriculum on curriculum-referenced tests of fraction computation skills ($ES M = 1.09$; Kelly et al., 1986; Kelly et al., 1990).

While implementing gradually sequenced instruction, two studies (Hughes, 2011; Watt, 2013) combined the instructional components of concrete and visual representations and sequence of examples. The sequence of examples in the graduated instructional sequence included concrete representations such as laminated construction paper circles cut into fractional pieces and strips, semi-concrete representations such as drawings of objects referring to fractions, and abstract fractions with no support materials. Moreover, teachers explicitly demonstrated target skills (e.g., equivalent fractions, adding and subtracting fractions) by modeling how to think aloud and monitored students’ responses. In

this way, the treatment group, receiving concrete–semi-concrete–abstract instruction, significantly outperformed the control group ($ES M = 1.73, p < .001$).

Concrete and visual representations; explicit, systematic instruction; range and sequence of examples; and heuristic strategies. One single-subject study (Reneau, 2012) implemented CRA with the mnemonic LISTS strategy when teaching word problems with fractions to five fifth graders struggling to learn mathematics. Reneau combined virtual manipulatives and schema-based instruction when providing gradually sequenced instruction with systematic instruction (i.e., modeling, guided, independent practice). Results indicated that all students made some gains on their probes with a PAND mean of 91%. Tau-U values ranged from .74 to 1.00, $p < .05$ for all students. The weighted average across five students was .83 ($CI_{.95} = [0.54, 1.13]$).

Heuristic strategies. Two single-subject studies (Joseph & Hunter, 2001; Test & Ellis, 2005) implemented heuristic strategies for students in resource rooms. Joseph and Hunter applied cue cards to improve the number of fraction problems calculated correctly for three eighth-grade students with LD. The PANDs of three students were high ($M = 97%$). Tau-U values for three students, respectively, were 1.00, .93, and .90, $p < .001$ for all students ($CI_{.90} = [0.40, 1.60]$, $[0.47, 1.38]$, and $[0.50, 1.31]$). The result for the weighted average across all students was also positive (Tau-U = .94, $CI_{.95} = [0.60, 1.28]$). Specifically, the high-average planner (Rob) and the average planner (Nick) showed the most stable performance, with a mean increase of 71% and 54%, respectively, from baseline to intervention. The below-average planner (Mark) fluctuated in performance, with a mean increase of 44% from baseline to intervention.

In another study, Test and Ellis (2005) implemented a self-regulatory mnemonic strategy for fraction computation problems (Look at the sign and denominator. Ask yourself the question. Pick your fraction type. Type 1. Type 2. Type 3. Identify denominator. Divide denominator. [LAP]). Three pairs of students' PAND mean on LAP Fraction intervention test was 100%. Tau-U values for all three pairs were 1.00, $ps < .001$. The result for the weighted average across all three pairs was also positive (Tau-U = 1.00, $CI_{.95} = [0.64, 1.36]$, $[0.65, 1.35]$, $[0.62, 1.38]$). Results indicated a functional relationship between implementing LAP Fractions and students' application of the strategy to adding and subtracting fractions. Five of 6 students mastered both skills and maintained their performance over 6 weeks.

Heuristic strategies and visual representations. One study (Lambert, 1996) implemented heuristic strategies through visual representations. Teachers taught eight steps of a cognitive strategy to 9th- through 12th-grade students with LD by using a cue card and visualizing problems. The eight

steps were read (for understanding), paraphrase (your own words), visualize (a picture or a diagram), state the problem, hypothesize, estimate, calculate, and self-check. Although there was a fairly large ES of .61, no significant difference was found on a problem-solving fraction test between the cognitive strategy group and the textbook instruction group.

Heuristic strategies, visual representations, and explicit, systematic instruction. One single-subject study (Shin, 2013) implemented cognitive and meta-cognitive strategies with virtual manipulatives when teaching word problems containing fractions and multiplication. Practice was systematically provided with video-based modeling and guided practice. Three students who used *Fun Fraction*, a web-based strategic, interactive computer application improved on their instructional probes (PAND $M = 82%$). Tau-U values ranged from .56 to 1.00 where only Alec's data were statistically significant (Tau-U = 1, $p < .05$, $CI_{.90} = [0.34, 1.66]$). The weighted average across three students was .74 ($CI_{.95} = [0.31, 1.18]$).

Use of real-world problems and concrete and visual representations. Three studies (Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2002) included real-world problems combined with concrete and visual representations. These studies implemented videodisc software that included pictorial representations in combination with video-based anchored instruction (AI; Bottge, 1999; Bottge & Hasselbring, 1993) or enhanced anchored instruction (EAI; Bottge et al., 2002). AI involves situating learning in a problem-based context and engages all students in learning through a variety of activities. An "anchor" might be a video, story, or problem that has meaning for students (for more information, see <http://viking.coe.uh.edu/~ichen/ebook/et-it/ai.htm>). EAI extends the idea of AI using technology and hands-on projects to facilitate the learning of students with LD and/or emotional disabilities (for more information, see http://team.wceruw.org/about_TEAM.html).

In Bottge (1999) and Bottge and Hasselbring (1993), LA secondary students and students with LD received video-based AI via contextualized problems, titled *The 8th Caller* and *Bart's Pet Project*, and were encouraged to share their solutions to the challenge problems from the video. The studies implemented these short (i.e., 8-min) video anchors when introducing fraction concepts. The AI allowed students to solve real-world problems in authentic contexts embedded in the short videos. Students were also encouraged to discuss and collaborate to solve subproblems within the videos. The AI group outperformed the word problem instruction group on a contextualized test, including adding and subtracting with fractions ($ES = 1.67$ and $1.11, p < .01$); however, there was only a small effect on the WP test ($ESs = .26$ and $.39$) and no effect on the computation test ($ES = -0.46$).

In the other study (Bottge et al., 2002), seventh graders with LD received EAI. The instruction consisted of an 8-min contextualized video problem, titled *Fraction of the Cost*, and a hands-on problem of planning and building wooden benches. *Fraction of the Cost* provided more challenging problems than *Bart's Pet Project* (Bottge et al., 2010). Instruction took place in either the general mathematics classroom or the technology education classroom. Consistent with previous findings, students in the Bottge et al. (2002) study who received EAI outperformed students who received WP instruction on the contextualized problem test ($ES = 0.92$). Three of four students with disabilities receiving EAI showed modest improvement on the fraction WP test. However, in general, there were no intervention effects on computation ($ES = -0.89$) or WP ($ES = -0.14$) tests.

Use of real-world problems, concrete and visual representations, and explicit, systematic instruction. Two studies (Bottge et al., 2014; Bottge et al., 2010) included real-world problems combined with concrete and visual representations and explicit, systematic instruction. Middle school students with LD and other disabilities who received EAI with explicit instructional modules made greater gains on their fraction tests than students in the EAI-only group (Bottge et al., 2010) or the in business-as-usual group (Bottge et al., 2014). In Bottge et al. (2010), a combination of explicit computer-based instruction (i.e., *Fractions at Work*) and EAI had a large effect on students' performance on the fraction computation test ($ES = 1.04$). However, there were no group differences between the combination instruction group and the EAI group on both the researcher-developed problem solving ($ES = 0.10$) and *Iowa Tests of Basic Skills* (ITBS) Problem Solving and Data Interpretation ($ES = 0.11$). Bottge et al.'s (2014) results concurred with those of their previous studies. That is, middle school students with disabilities in the EAI group significantly outscored their peers in the business-as-usual instruction on fraction computation test ($ES = 1.19$). Moreover, there were positive effects of intervention on ITBS computation ($ES = 0.59$) and even a researcher-developed problem-solving ($ES = 0.48$) test.

Discussion

Understanding and mastery of fractions is essential prerequisite knowledge for algebraic instruction (NMAP, 2008). Underscoring the importance of such knowledge, the CCSSM (NGAC & CCSSO, 2010) for Grades 3 through 5 stipulate fraction concepts and skills to be taught. Thus, it is clear that if they are to succeed in school and beyond in the 21st century, fraction instruction is critical for all students, including students struggling to learn mathematics.

Features of Fraction Interventions

The findings of the present study shed light on current as well as best practices and, therefore, are critical for ensuring that all students, including those with disabilities, acquire necessary skills and competencies. With regard to instructional opportunities to practice fraction interventions, researchers (e.g., Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008) have emphasized the importance of sufficient time for practice during mathematics interventions, suggesting that students' instructional time and opportunities to practice are related to instructional effects. In this synthesis, only 7 out of 17 studies implemented interventions longer than 10 sessions, and 3 did not report the total amount of time spent on instruction. Bottge et al. (2014) devoted approximately 4,230 to 5,640 min to instruction. Not surprising, Bottge et al.'s (2014) study showed higher positive outcomes than the previous study by Bottge et al. (2010), in which 1,200 to 1,320 min were devoted to instructional time. This finding suggests intensifying instruction by increasing time (duration of the study) can be beneficial for struggling students to better learn fraction concepts (National Center on Intensive Intervention, 2013).

CCSSM

We extended the work of Misquitta (2011) by examining connections to the CCSSM for each study; this aspect of the present article is important because of the extent to which the standards are now implemented in the United States. According to CCSSM (NGAC & CCSSO, 2010), 43 states, the District of Columbia, 4 territories, and the Department of Defense Education Activity have not adopted the *Common Core State Standards*. Thus, educators are seeking connections between the CCSSM and the intervention programs they use. To this end, we analyzed targeted fraction concepts and mathematical practice for each grade level.

Regarding the *CCSSM Standards for Mathematical Content* (NGAC & CCSSO, 2010), 12 of the 17 studies (e.g., Bottge et al., 2014; Bottge et al., 2010) included addition and subtraction of fractions with like and unlike denominators; 8 (e.g., Shin, 2013; Watt, 2013) targeted multiplication of fractions. It is important to strengthen the understanding of the inverse relationships between multiplication and division of fractions, including partitioning concepts, in middle school mathematics (Siemon, 2003). However, many studies did not focus on multiplication and division of fractions; none of the studies included negative fractions; and none addressed how to represent fractions on a number line by linking conceptual and procedural knowledge (NMAP, 2008; Siegler et al., 2010). Considering that these are the foundational skills related to middle and high school CCSSM, secondary students struggling to learn mathematics should learn these various fraction concepts

and skills (Powell, Fuchs, & Fuchs, 2013). In addition, according to the recommendations in an Institute of Education Sciences practice guide (Siegler et al., 2010), number lines (a) help students understand that fractions are also numbers in the number system, (b) promote students' understanding of fraction equivalence and negative numbers, and (c) allow students to mathematically translate among rational numbers of common fractions, decimals, and percentages. Thus, the lack of number line use in the included studies is problematic in terms of helping students make conceptual connections.

Regarding the *CCSSM Standards for Mathematical Practice* (NGAC & CCSSO, 2010), modeling was the most frequently observed practice. Fifteen studies included visual representations (e.g., small circle, rectangular, fraction pies). In addition, 8 of the 17 studies encouraged discourse among students and teachers for further understanding of mathematics. This is an important finding because rather than using rote algorithms, the application of meaningful mathematical processes through discussion in a group is an essential goal of mathematics instruction (Woodward, 2008). Nevertheless, in the Bottge et al. (2002) study, students who received low levels of attention during instruction performed poorly on fraction tests, even if they received effective fraction interventions through mathematical representations and discourses. This finding indicates that reform-based mathematics instruction, which highlights whole-class discussion, may be challenging and impose a high degree of cognitive load for struggling learners (Baxter, Woodward, Voorhies, & Wong, 2002). Thus, mathematics instruction for struggling learners should be *explicit* coupled with the use of representations and student dialogue even for developing conceptual understanding and student-led mathematical reasoning activity in class.

Instructional Components and Effects of Interventions

Moreover, rather than focusing on the effects of interventions according to their types (Misquitta, 2011), the present study analyzed how each intervention consisted of various instructional components. The most commonly used instructional component was concrete and visual representations. The studies showed large effects for the application of visual representations (CCSSM MP 4; NGAC & CCSSO, 2010). Many researchers recommend the use of visual representations (e.g., number lines, fraction bars) to encourage students' conceptual understanding of fractions beyond computational procedures (NMAP, 2008; Siegler et al., 2010). Therefore, it is not surprising that most studies ($n = 15$) included concrete and visual representations within their interventions. With regard to the effect of concrete versus visual representations, Butler et al. (2003) found that students in a CRA condition performed better than students

in a RA condition. Martin and Schwartz (2005) concurred with this finding, reporting that the manipulation of physical objects (e.g., tile and pie pieces) was more beneficial to students for developing fraction concepts of partitioning than drawing pictures by pencil.

Ten of the 17 studies (e.g., Bottge et al., 2014; Bottge et al., 2010; Hunt, 2014; Watt, 2013) that used concrete and visual representations in combination with explicit, systematic instruction showed highly positive outcomes on fraction concepts and skills. This finding suggests that the features of explicit, systematic instruction—including explicit modeling of step-by-step strategies, sufficient practice, and cumulative reviews with feedback (NMAP, 2008)—enhance instruction in combination with visuals. Furthermore, it corroborates the results of a meta-analysis by Gersten et al. (2009) in which the use of visual representations in combination with other instructional components led to greater effects than the use of visuals alone during instruction.

Five studies (Joseph & Hunter, 2001; Lambert, 1996; Reneau, 2012; Shin, 2013; Test & Ellis, 2005) implemented heuristic strategies by promoting mathematical verbalization with a “think-aloud” strategy or in combination with virtual manipulatives, leading to improved problem-solving and fraction computation skills. Other studies supported the use of a heuristic strategy (Gersten et al., 2009), noting a significant effect for students with LD in emphasizing student verbalizations and the reflective process of their mathematical problem solving.

Finally, the use of real-world problems, connecting fraction skills to contextualized problems via video-based AI or EAI, resulted in improvements on contextualized tests for students struggling to learn mathematics but not on fraction computation and problem-solving tests (Bottge, 1999; Bottge & Hasselbring, 1993; Bottge et al., 2002). This finding concurred with Misquitta's (2011) synthesis, which showed that implicit instruction did not improve struggling learners' performance on fraction computation tasks and indicated a need for explicit teaching for students. However, when EAI was provided in combination with explicit, computer-based instruction, students with disabilities outperformed their control group on fraction computation and problem solving (Bottge et al., 2014; Bottge et al., 2010). These findings highlight the significant role of explicit instruction in delivering mathematics instruction to struggling learners.

Limitations and Future Research

Several limitations and suggestions for future research emerge from the findings of the present synthesis. First, a limited number of studies (17 meeting the inclusion criteria) and participants (805 students) were examined. In the future, more studies should focus on improving fraction

concepts and skills of struggling learners. Second, most of the studies ($n = 15$) included secondary school students; only 2 included elementary school students. Thus, future research should include more elementary school students. Third, the focus of fraction concepts and procedures was limited. Regarding the *CCSSM Standards for Mathematical Content* (NGAC & CCSSO, 2010), 12 of the 17 studies mainly focused on adding and subtracting fractions.

Considering the significance of fraction curricula as the foundational content for algebra (NMAP, 2008) in middle and high school, in the future, more studies should incorporate mathematics interventions that address multiplicative reasoning and the inverse relationship with division of fractions, including making a connection to algebraic problem solving. Last, the focus on various *CCSSM Standards for Mathematical Practice* (NGAC & CCSSO, 2010) was limited. In the future, studies need to implement diverse mathematical problem-solving methods beyond the use of mathematical modeling (MP 4) and problem-solving processes (MP 1). By connecting the missing mathematical practices (e.g., mathematical precision and use of structure), educators could investigate how different fraction concepts and skills are aligned with each practice.

Educational Implications for Instruction

Fractions are one of the critical topics that students must understand and master as a prerequisite for algebra instruction. There are too many students who struggle with concepts in fractions in the middle and high school levels. Without a solid foundation in understanding these concepts, students are doomed to continue struggling with higher level mathematics. Instruction must go beyond teaching “tricks” for solving equations with fractions and focus on remediating misconceptions. Thus, importantly, this synthesis yielded three critical implications for practice. First, fraction instruction for students struggling to learn mathematics is challenging for teachers. Thus, teachers need to be familiar not only with the content knowledge of fractions but also with the features of fraction interventions, including evidence-based instructional components. Gersten et al. (2009) found strong evidence for the instructional features of explicit, systematic instruction including systematic teacher modeling, profound practice opportunities, and cumulative review. Second, teachers should also consider that not all students demonstrate the same degree and rate of improvement. Some students struggling to learn mathematics need more intensive instruction and guidance than others. One way to intensify instruction, which was evident in findings from the Bottge et al. (2014) study, is to provide more time with key concepts about fractions using a combination of EAI and systematic instruction. With these key implications in mind, the likelihood of improving fraction instruction for students struggling to learn mathematics is

promising. Third, teachers and researchers can address various mathematical practices associated with the CCSSM (NGAC & CCSSO, 2010) for teaching fraction concepts and skills. By addressing mathematical “processes and proficiencies,” teachers can help students struggling to learn mathematics to improve their conceptual and procedural understanding of fractions that are taught across the grade levels. In that way, students can potentially generalize their own mathematical reasoning to problem solving and other mathematical topics, such as ratios and proportions, in further, more advanced mathematics instruction.

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